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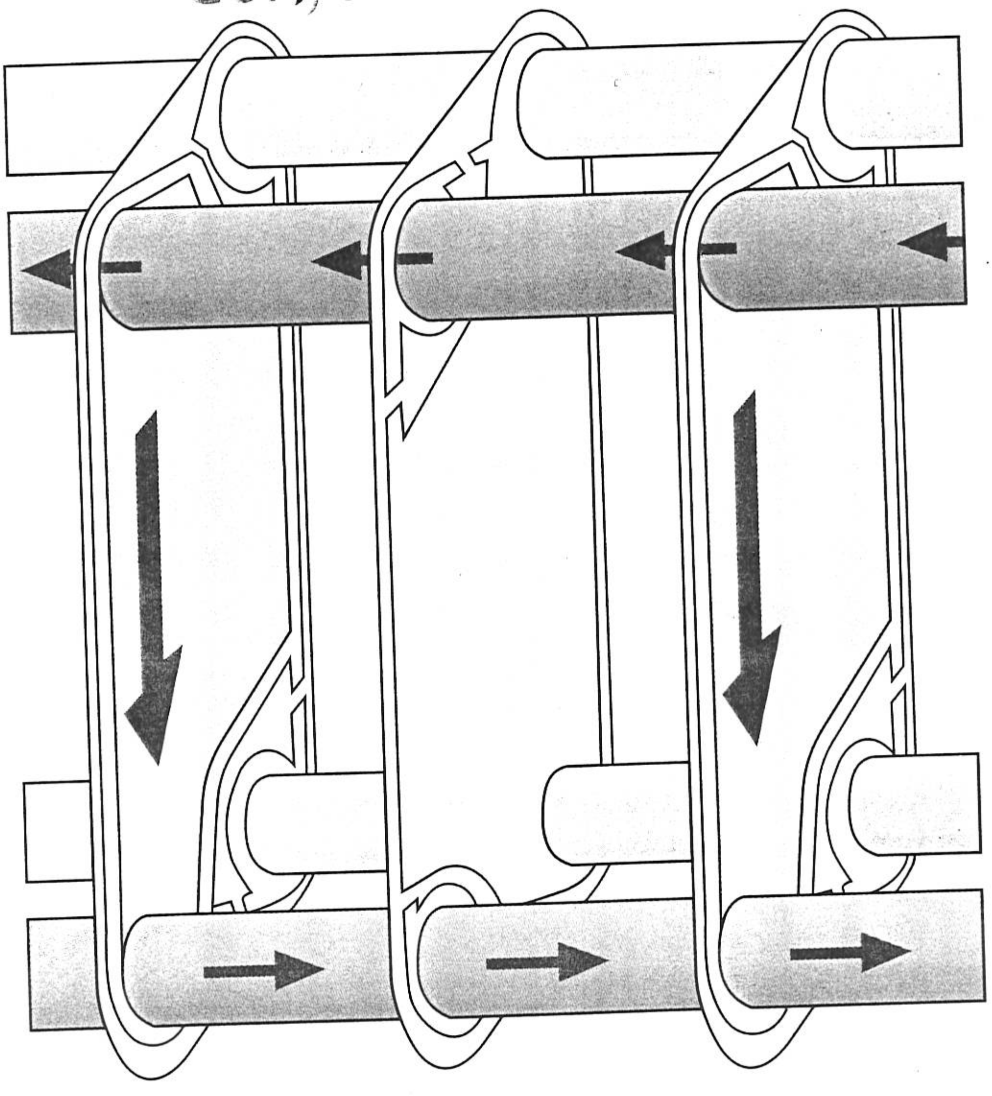

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AN INVESTIGATION OF HEAT TRANSFER BY WAVES IN A ROTATING FLUID ANNULUS BLOCKED BY A RADIAL BARRIER.

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Several mechanisms combine to keep the total fluid heat transport in a differentially heated rotating fluid annulus that is blocked by a radial barrier largely independent of rotation rate. Of particular interest are the time-dependent waves which appear at certain values of the two principal dimensionless parameters for the system, $\Theta \equiv g\alpha\Delta T d/(\Omega^2(b-a)^2)$ and $\tau \equiv 4\Omega^2(b-a)^5/(\nu^2 d)$. The mechanism causing these waves is not known. By sloping the base of the convection chamber it has proved possible to suppress the formation of the waves. This has allowed their heat transport characteristics to be inferred and provides strong evidence that they arise by a process known as "sloping convection" or baroclinic instability.

Keywords: Rotating fluid, Annular geometry, Heat transfer, Radial barrier, Baroclinic waves.

INTRODUCTION

The study of heat transfer in rotating fluids can contribute to the understanding of fluid motions in geophysical systems (e.g. atmospheres and oceans). Basic fluid processes can be elucidated by studying the rotating fluid annulus (Figure 1), which is an idealised representation of such systems, as well as being a system of considerable interest in its own right. The total radial fluid heat transport, Q' though a differentially heated annulus of liquid rotating with uniform angular velocity, Ω is, in general, strongly dependent upon Ω, as well as the imposed temperature difference ΔT, the convection chamber geometry and fluid properties. For an annulus with no barrier $Q'(\Omega)$ decreases from $Q'(\Omega=0)$ with increasing Ω (Hide, (1)). The introduction of a thin, rigid, thermally insulating radial barrier into the convection chamber of an annulus of uniform depth causes Q' to become largely independent of Ω (Bowden and Eden (2) and Rayer (3)) (Figure 2a). The fluid temperature difference observed across the insulating barrier, ΔT_B is associated with fluid heat advection, Q'_A, by a process known as azimuthal geostrophic balance (3). Replacing the insulating barrier with a conductor, in an attempt to reduce ΔT_B and thereby effect Q'_A has little effect on heat transport, in fact leading to a slight increase in ΔT_B (Rayer, (4)).

For azimuthal geostrophic balance to keep Q'_A independent of Ω it requires that $\Delta T_B \propto \Omega$. The maximum ΔT_B seen in the uniform depth experiments is about 25% of ΔT (Figure 2b), however Q' continues to remain largely independent of Ω over a larger range of Ω than can be accounted for by azimuthal geostrophic balance alone (Figure 2a).

When the dimensionless parameters $\Theta \equiv g\alpha\Delta T d(\Omega^2(b-a)^2)^{-1} \leq 0.5$ and $\tau \equiv 4\Omega^2(b-a)^5(\nu^2 d)^{-1} \geq 10^7$

Parameter	Symbol	Value
Radii of inner & outer cylinder	a, b	2.5×10^{-2} & 8.0×10^{-2} m
Depth of chamber	d	14.0×10^{-2} m (Δ=0)
		10.6×10^{-2} m (mean depth, $\Delta\neq0$)
Height of base above z=-d/2	Δ	0-51×10^{-3} m
Angular velocity	Ω	0.0-5.0 rad.sec^{-1}
Mean fluid temperature	T_0	293 K
Applied temperature difference	ΔT	4 K
Kinematic viscosity of fluid	ν	1.8×10^{-4} m^2.sec^{-1}
Specific heat capacity of fluid	C_P	3.84×10^3-3.85×10^3 J.kg^{-1}.K^{-1}
Mean density of fluid	ρ_0	1.045×10^3-1.088×10^3 kg.m^{-3}
Expansion coefficient of fluid	α	3.03×10^{-4} K^{-1}
Thermal conductivity of fluid	k	5.2×10^{-1} W.m^{-1}.K^{-1}

Table 1. Definition and ranges of experimental parameters.

(see Table 1 for the nomenclature used) time-dependent aperiodic waves are seen in the flow. The values of Θ and τ for which waves are seen in the experiments include the range of Ω for which ΔT_B is not proportional to Ω, and where azimuthal geostrophic balance cannot account for the total fluid heat advection. It therefore seems reasonable to investigate whether the waves carry heat, particularly as the mechanism causing the waves has not been clearly identified. This paper reports the results of experiments carried out to determine whether the waves do transport significant quantities of heat, and the method used to examine this problem has yielded information about the fundamental processes responsible for the onset of these waves.

Previous work (Hide and Mason (5) and Hide (6)) has shown that sloping bases can be effective in suppressing the formation of baroclinic waves in unblocked annulus experiments. Baroclinic waves arise by a process known as sloping convection, where fluid particles release energy by moving on surfaces which are tilted at a relatively small angle to the horizontal that is comparable with, but less than, the slope of the isotherms. Increasing Ω tends to constrain fluid particles to move on surfaces parallel to the lid and base of the apparatus, as Mason (7) puts it, "At more rapid rotation rates the vertical stiffness of the fluid increases and the presence of the boundaries is felt throughout the interior of the fluid". Baroclinic waves (sloping convection) can only occur when fluid particles move at certain well defined angles relative to the geopotential and isothermal surfaces in the fluid. Sloping bases, by constraining fluid particles to move in planes either suitable or unsuitable for sloping convection can either encourage or inhibit the formation of baroclinic waves.

In this case the aim was to inhibit sloping convection to see what effect this would have on the time-dependent waves seen in the blocked annulus system. If the waves were suppressed their contribution to the overall heat transport could be deduced by comparison between sets of measurements.

APPARATUS DESCRIPTION

The annular convection chamber was formed by trapping fluid between two co-axial cylinders and a thermally insulating rigid lid (at $z=d/2$) and base (at $z=-d/2+\Delta$), where z is the vertical direction in cylindrical polar coordinates, (r, ϕ, z). Initial measurements had been carried out with a convection chamber of uniform depth ($\Delta(r, \phi)=0$) (3). For the additional measurements reported here, $\Delta\neq0$ as the base sloped in both the radial and azimuthal directions according to $\Delta(r, \phi)=25.0\times10^{-3}(r-a)(b-a)^{-1}+6.5\times10^{-3}\phi\pi^{-1}+19.5\times10^{-3}$ (m).

Differential heating was provided by maintaining the cylinders at two different constant temperatures, T_a and T_b (see Figure 1), with $T_b>T_a$. The chamber was placed upon a turntable so that its central axis of symmetry coincided with the axis of rotation of the turntable, and could be rotated over a range of uniform angular velocities, $\underline{\Omega}=(0, 0, \Omega)$. The dimensions of the annulus, fluid properties and the ranges of certain other experimental parameters are given in Table 1.

Measurements of Q' were made by calorimetry. Temperature measurements, taken as a temporal mean, were made using a ring of thermocouples equally spaced around the azimuth, at mid-depth and mid-radius. Fluid motions were visualised by using neutrally buoyant tracer particles of diameter 600-700μm, and used to infer fluid velocities and the three-dimensional structure of the flow.

EXPERIMENTAL RESULTS

The typical flow in the convection chamber has been described previously (3, 4). Fluid rises by the hot outer cylinder, moves radially across the chamber at the top, before sinking by the cool inner cylinder and returning to the outer cylinder at the bottom of the chamber. As Ω increases there is also a horizontal circulation, with the same sense as the background rotation, which is superimposed upon the radial over-turning cell. At higher Ω the time-dependent waves mentioned above were observed in the system with $\Delta=0$, however the sloping base prevented the formation of the waves over the entire range of parameters used $1.64\times10^{-2}<\Theta<\infty$, $0 < \tau < 1.46\times10^{8}$.

Heat Transport and Temperature Measurements

Plots of ΔT_B at mid-height and mid-radius are given in Figure 2(b). The results of the heat transport measurements are given in Figure 2(a). When $\Delta\neq0$ the heat transport was reduced, to a minimum of about 80% of the uniform depth case. Accompanying this there has been an increase in the maximum value of ΔT_B seen in the system.

DISCUSSION

The sloping base prevented the formation of the time-dependent waves so that the heat transport measurements, Q' in the system without waves could be compared with Q' when waves were present. The results provide strong evidence that the waves transport significant quantities of heat. Further, the waves were suppressed by suitably sloping the base of the annulus. Both of these are characteristics typical of baroclinic waves. Considering the manner in which sloping bases effect the flows (see the Introduction and (5-7)) the fact that the waves were suppressed when $\Delta\neq0$ provides compelling evidence that the waves are baroclinic in nature. In addition the values of Θ and τ at which the waves appear in the $\Delta=0$ case ($\Theta \leq 0.5$, $\tau \geq 10^7$) are similar to the values at which regular waves are seen in the unblocked annulus system ($\Theta \leq 2$, $\tau \geq 10^6$, private communication D. W. Johnson), supporting the view that the waves in the blocked system are baroclinic.

The sloping base also effected $\Delta T_B(\Omega)$, which was increased to a maximum value 2.5 times that of the $\Delta=0$ case. Two possible conclusions can be drawn from this. Firstly as the heat advection arising from the radial overturning cell associated with azimuthal geostrophic balance is proportional to ΔT_B (3, 4) this allows greater heat advection than would have been possible otherwise, perhaps partially making up for the heat that would have been transported by the waves. Secondly the mechanism which determines the maximum value of $\Delta T_B(\Omega)$ is not currently understood. It is possible that baroclinic waves, as well as transporting heat radially across the annulus, also transport heat azimuthally around the annulus, thereby acting to reduce ΔT_B. This idea is supported by the experimental observation that the waves tended to propagate azimuthally around the convection chamber. If sloping convection plays a role in setting the maximum value of ΔT_B then preventing it would allow ΔT_B to reach higher values than otherwise possible.

CONCLUSIONS

Total fluid heat transport measurements have been made for a differentially heated rotating fluid annulus which was blocked by a thermally insulating barrier and had a sloping base. The sloping base suppressed the formation of the time-dependent waves seen in a similar system of uniform depth. The sloping base also had the effect of reducing the heat transport and increasing the azimuthal temperature gradient (as measured by ΔT_B) compared with the uniform depth system.

The results provide strong evidence that the time-dependent waves seen in the uniform-depth system arise from a process known as "sloping convection" or baroclinic instability. They could also suggest that the waves play a role in limiting the maximum azimuthal temperature gradient seen in the uniform depth system.

REFERENCES

1 Hide, R., 1977, Experiments with rotating fluids, Q. J. Roy. Met. Soc., 103: 1-28.

2 Bowden, M and Eden, H.F., 1968, Effect of a radial barrier on the convective flow in a rotating fluid annulus, J. Geophys. Res., 73: 6887-6896.

3 Rayer, Q.G., 1992, An experimental investigation of heat transfer by large-scale motions in rotating fluids, D.Phil. thesis, Oxford University, 250 pages.

4 Rayer, Q.G., 1995, Thermal convection in a rotating fluid annulus blocked by a conducting barrier, IMechE Conference Transactions, 4th UK National Conference on Heat Transfer, Manchester, UK, ISBN 0-85298-950-4, p177-181.

5 Hide, R. and Mason, P.J., 1975, Sloping convection in a rotating fluid, Adv. in Phys., 24: 1, 47-100.

6 Hide, R., 1969, Some laboratory experiments on the free thermal convection in a rotating fluid subject to a horizontal temperature gradient and their relation to the theory of the global atmospheric circulation, The global circulation of the atmosphere, Ed. G. A. Corby, London, Roy. Met. Soc..

7 Mason, P.J., 1975, Baroclinic waves in a container with sloping endwalls, Phil. Trans. Roy. Soc. London, A, 278: 397-445.

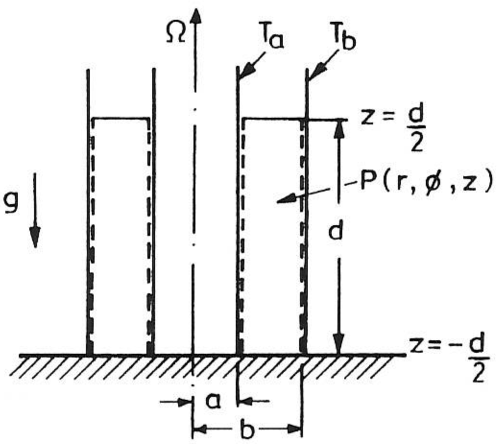

Figure 1. Diagram of fluid annulus. (r, ϕ, z) are cylindrical polar coordinates of a general point P, fixed in a rotating frame moving with the annulus which rotates uniformly with Ω rad.sec^{-1}.

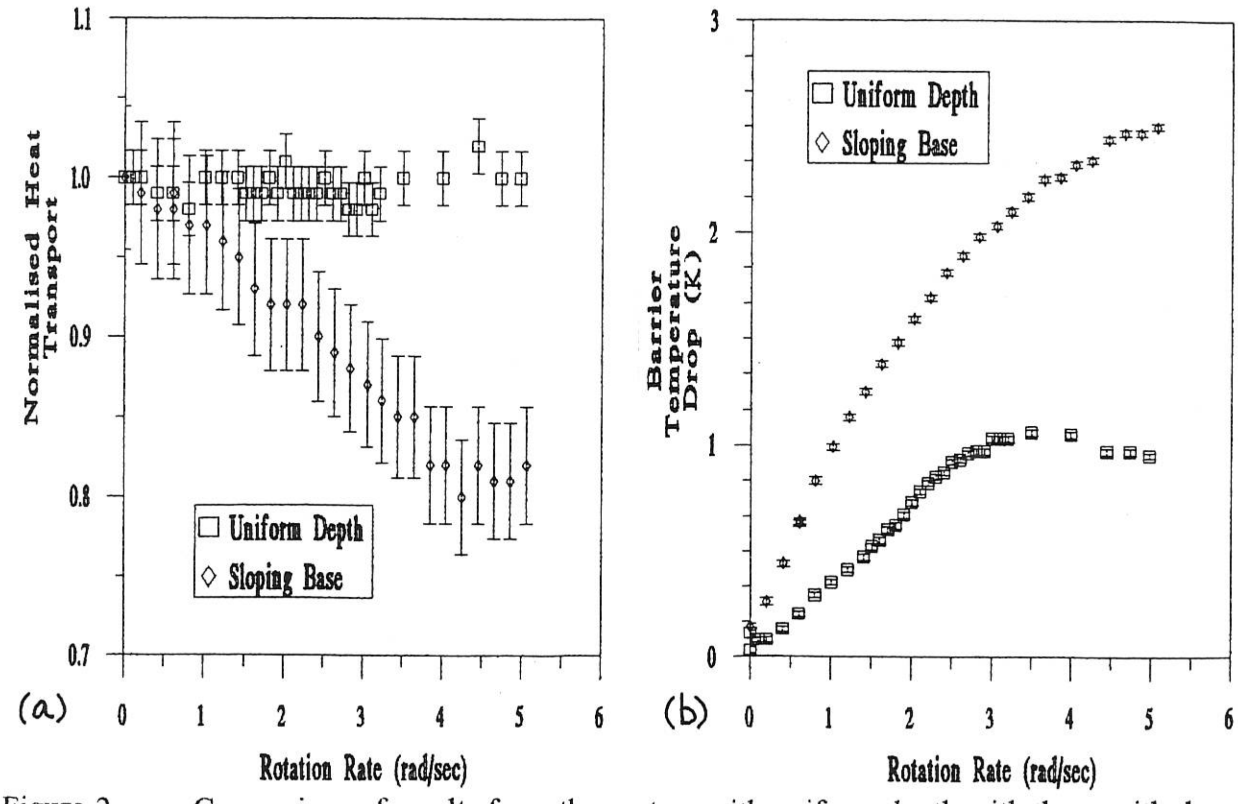

Figure 2. Comparison of results from the system with uniform depth with those with the sloping base plotted against Ω. (a) Normalised heat transport, $Q'(\Omega)/Q'(\Omega=0)$, (b) Barrier temperature drop, ΔT_B.