(1)

SIMULATION OF AXISYMMETRIC SOURCE-SINK FLOWS IN A ROTATING FLUID ANNULUS.

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PURPOSE OF WORK

(1) Validate Nuclear Electric Ltd CFD code FEAT (Finite-Element Analysis Toolbox) against rotating fluid flows *

(2) Explore some interesting physics.

→ Look for simplest possible rotating fluid flow hence :-

2D – axisymmetric flow

Isothermal – no buoyancy

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* ... and for me learn some CFD!

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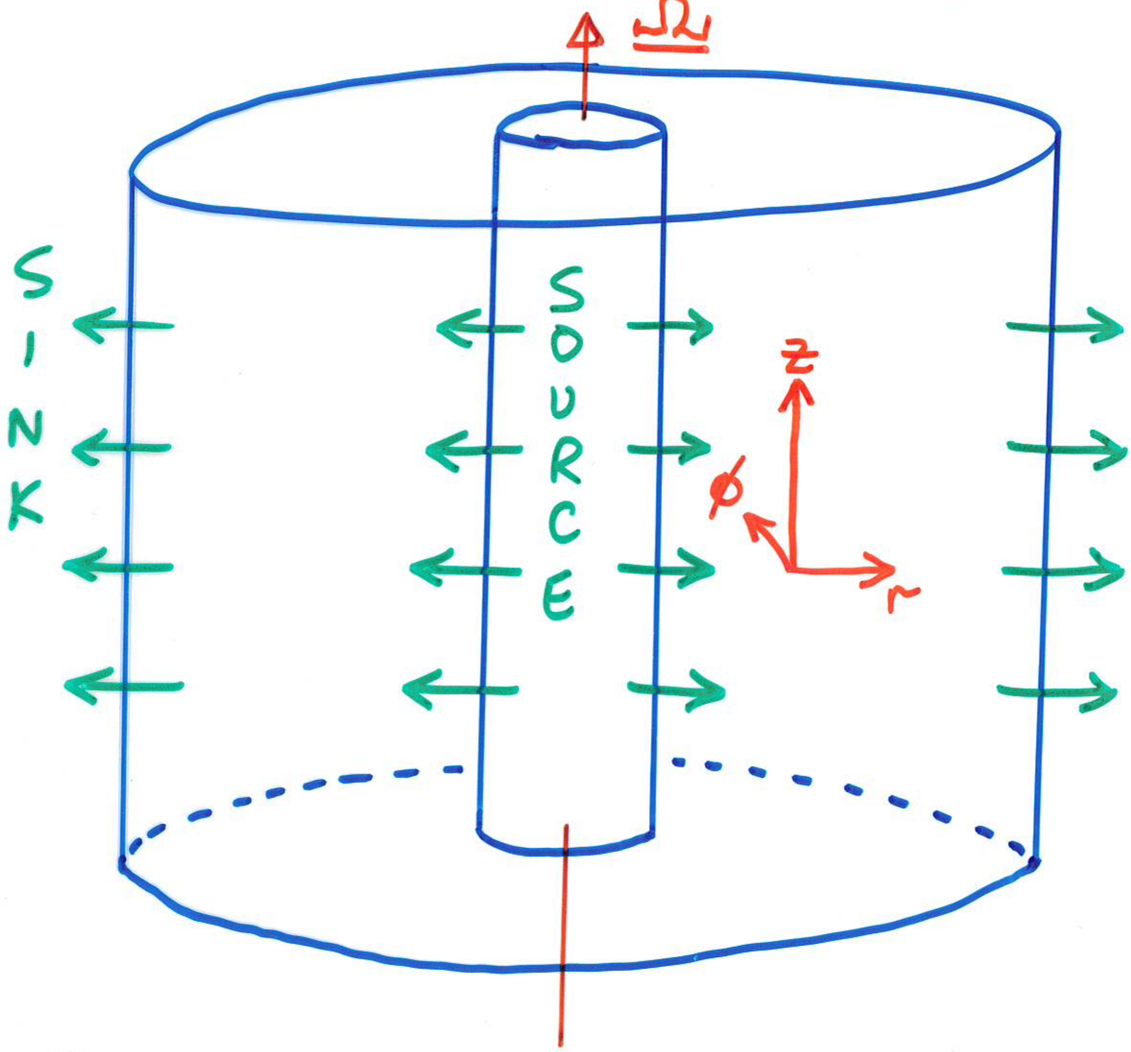
THE SYSTEM:

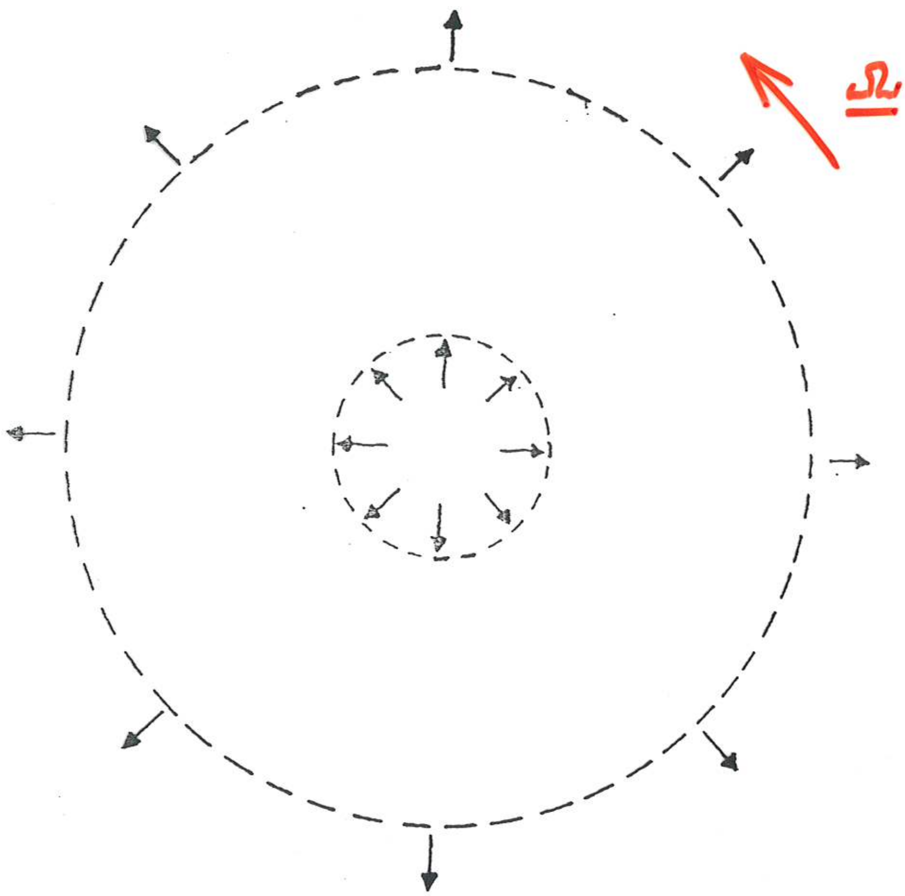
- Rotating fluid annulus
- Fluid pumped through radially

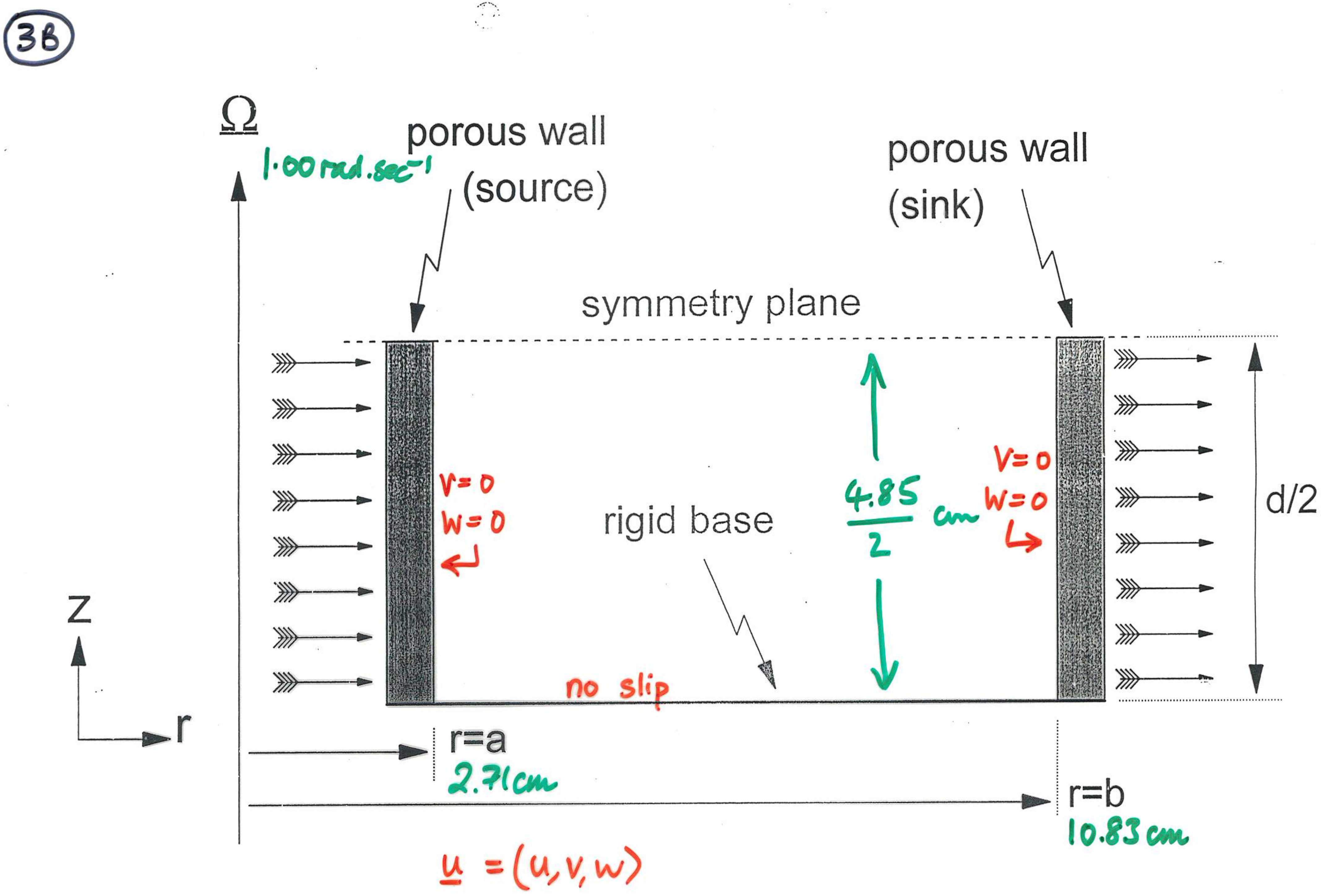


- Rigid, impermeable lid & base.
- Rigid porous cylindrical walls.
- Uniform rotation about central axis of symmetry.
- Walls, lid & base all rotate together.

(3A)

THE SYSTEM (from above)

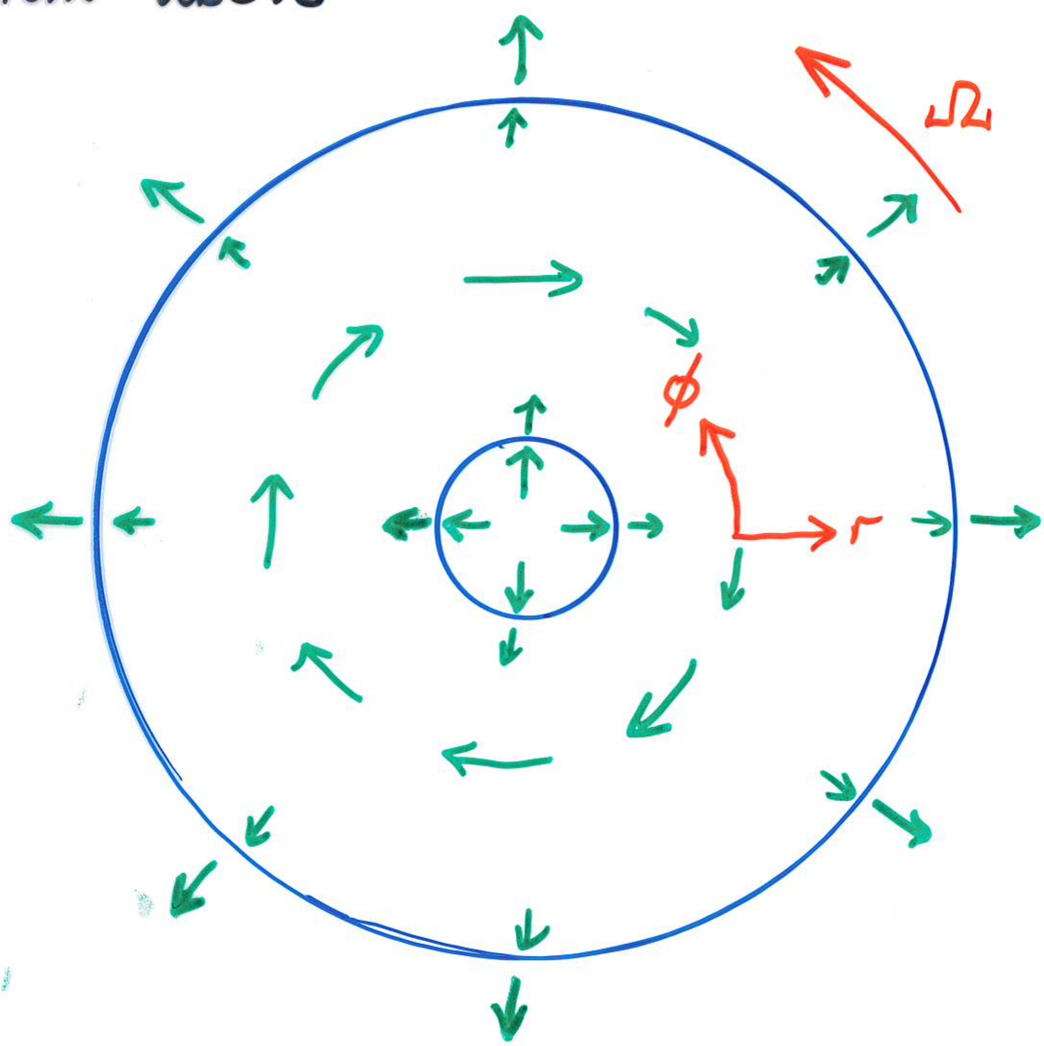



3B
Ω
1.00 rad.sec⁻¹
porous wall
(source)
porous wall
(sink)
symmetry plane
V=0
W=0
V=0
W=0
rigid base
4.85/2 cm
d/2
no slip
Z
r
r=a
2.71cm
r=b
10.83 cm
u = (u, v, w)

⑥

THE FLOW :

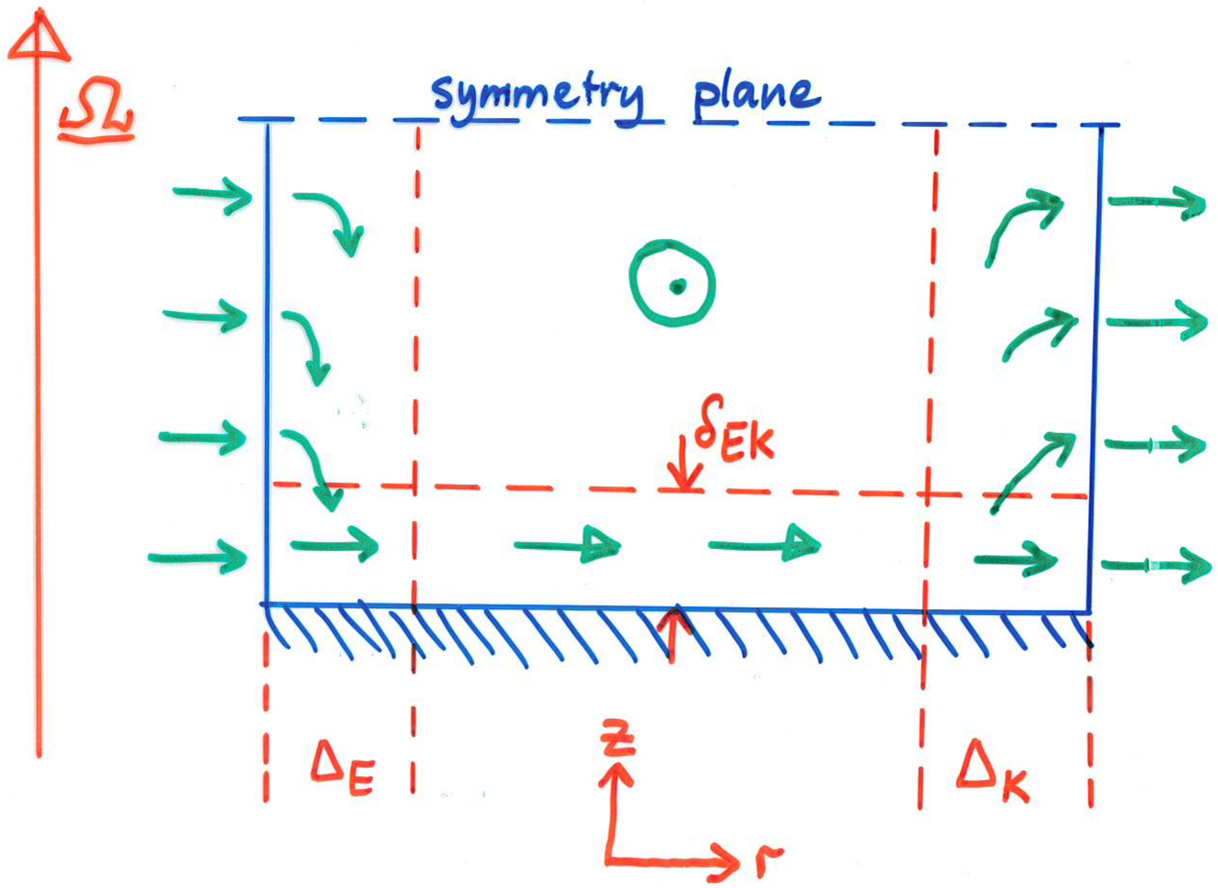
From above



Pumped fluid feels the effect of Coriolis acceleration $-2\,\underline{\Omega} \times \underline{u}$ and turns to the right in the fluid interior.

THE FLOW

In r-z plane.



- Azimuthal flow in fluid interior
- Radial flow in endwall boundary layers – Ekman, thickness $\delta_{EK} = 3\sqrt{\frac{\nu}{\Omega}}$
- Vertical flow in side-wall boundary layers : SOURCE layer thickness Δ_E

  SINK layer thickness Δ_K

PREVIOUS WORK.

- Experiments (Hide 1968, Bennetts & Jackson 1974)

  Azimuthal flow profiles

  Estimates of Δ_E & Δ_K

- Theory (Hide 1968, Barcilon 1970, Bennetts & Hocking 1973)

  Predictions of Δ_E & Δ_K

- A single computational study (Bennetts & Jackson 1974)

  Limited number of cases

  Flow profiles

  Spot-checks relative Δ_E & Δ_K

SIDE-WALL BOUNDARY LAYERS.

Typically
- Fix rotation rate, Ω
- Vary total flow through system Q.

Dimensionless Numbers

Rossby N° $= \dfrac{V}{\Omega d}$, Ekman N° $= \dfrac{\nu}{\Omega d^2}$

Linear Regime

$Ro \ll Ek^{1/4} \ll 1$ "Low" Q.

$\Delta_E, \Delta_K \sim \Delta_{Stewartson} = \ell Ek^{1/4}$

Can neglect inertial effects.

Non-Linear Regime

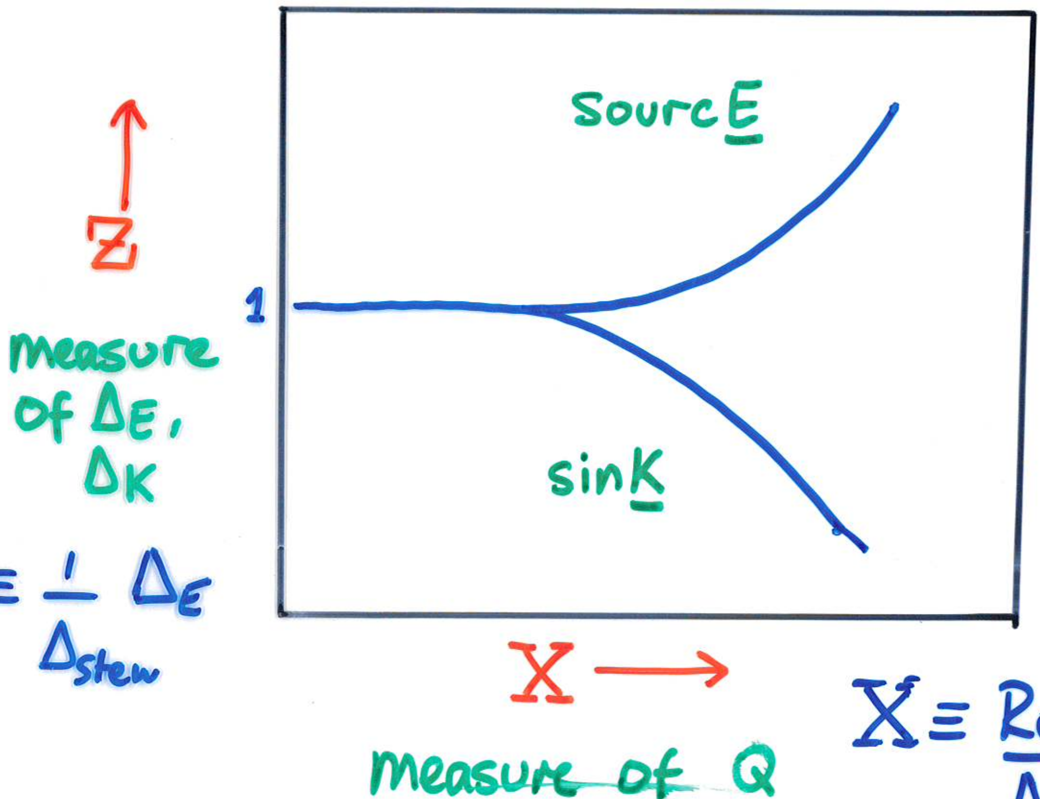
$1 \gg Ro \gg Ek^{1/4}$ "High" Q.

$\Delta_E \longrightarrow O(Ro)$ Thickens

$\Delta_K \longrightarrow O(Ek^{1/2} Ro^{-1})$ Thins.

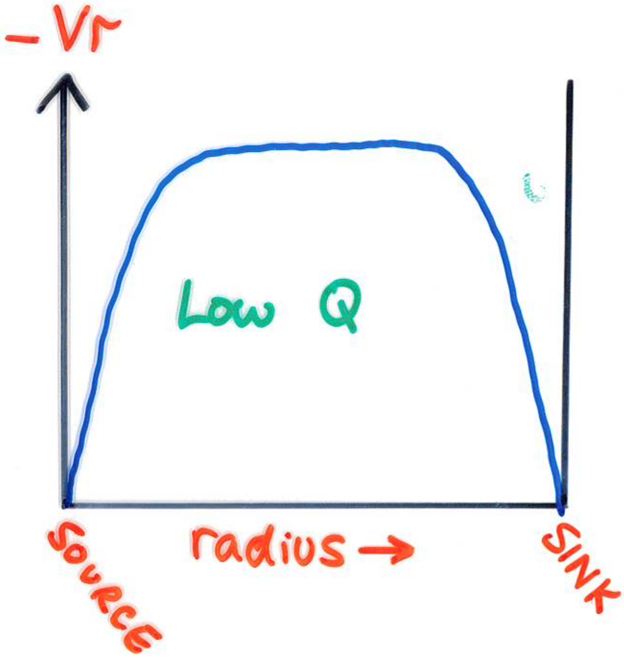
Azimuthal flow profile "skews"

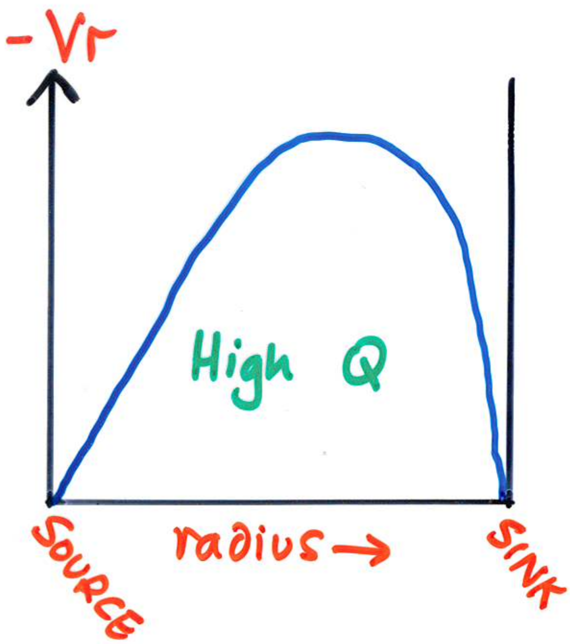
SOURCE & SINK LAYERS.

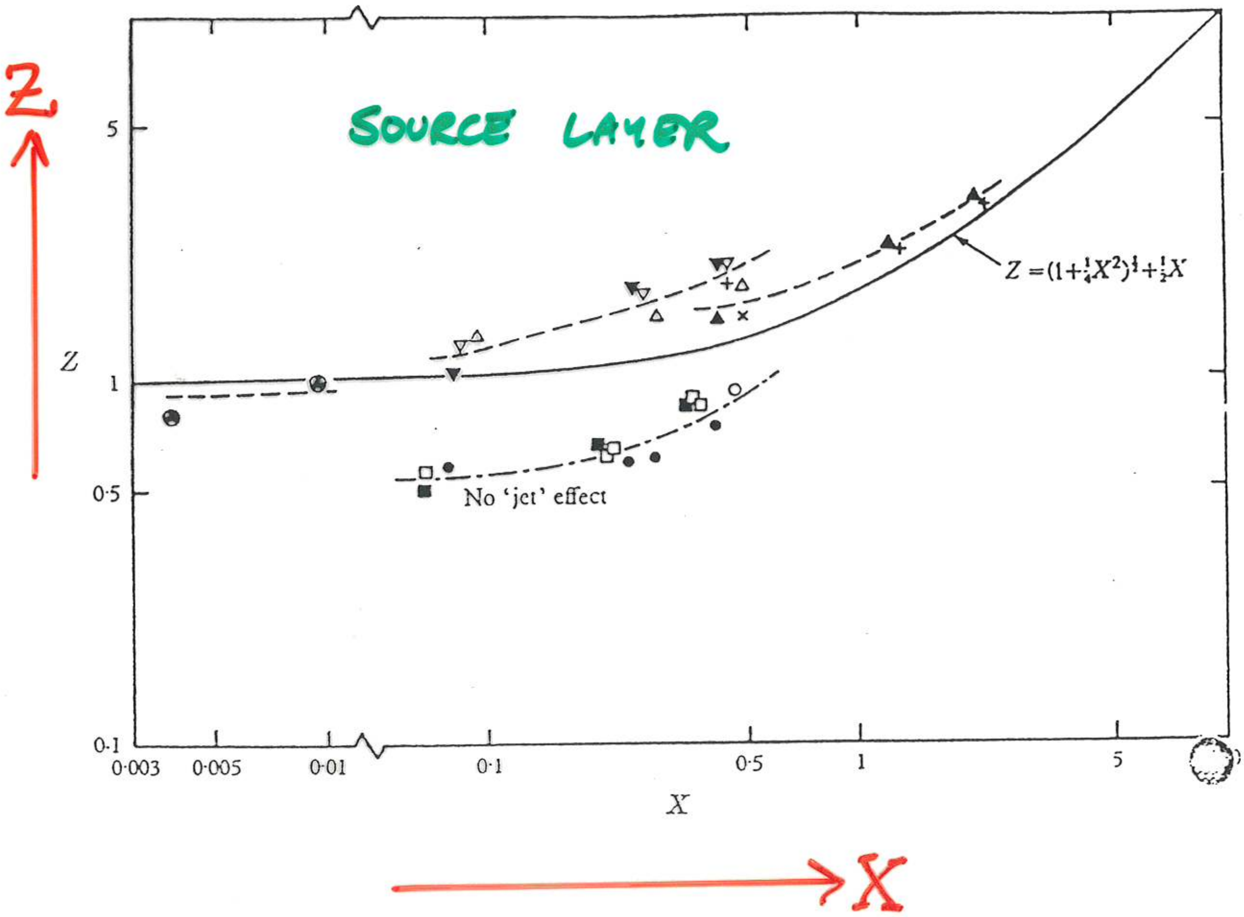


$$Z_E \equiv \frac{1}{\Delta_{stew}} \Delta_E$$

$$X \equiv \frac{Ro\,L}{\Delta_{stew}} \sim Ro\,Ek^{-1/4}$$







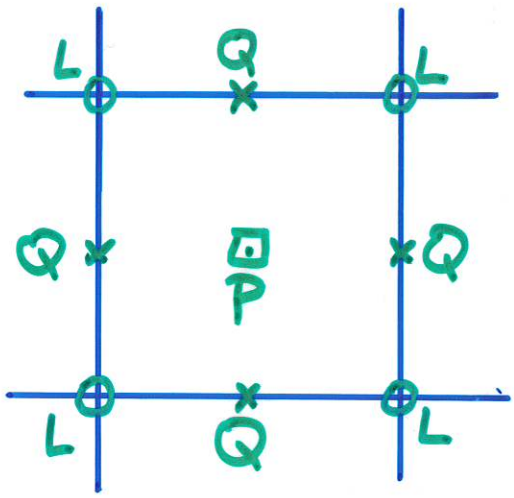
<u>Hide (1968)</u>

Experimental determinations of source layer thickness.

FEAT.

- Finite - Element
- Quadratic Interpolation
- Newton - Raphson solver (implicit)

MESH.



Linear interpolation – L-nodes

Quadratic interpolation – L + Q nodes

$\underline{u}$, T, ρ, ν etc. on L + Q nodes

Pressure on L + P nodes to improve representation of continuity equation.

FEAT (cont.)

Finite Element:

Galerkin Finite-Elements so:-

- no built-in upwinding
- centred-type scheme.

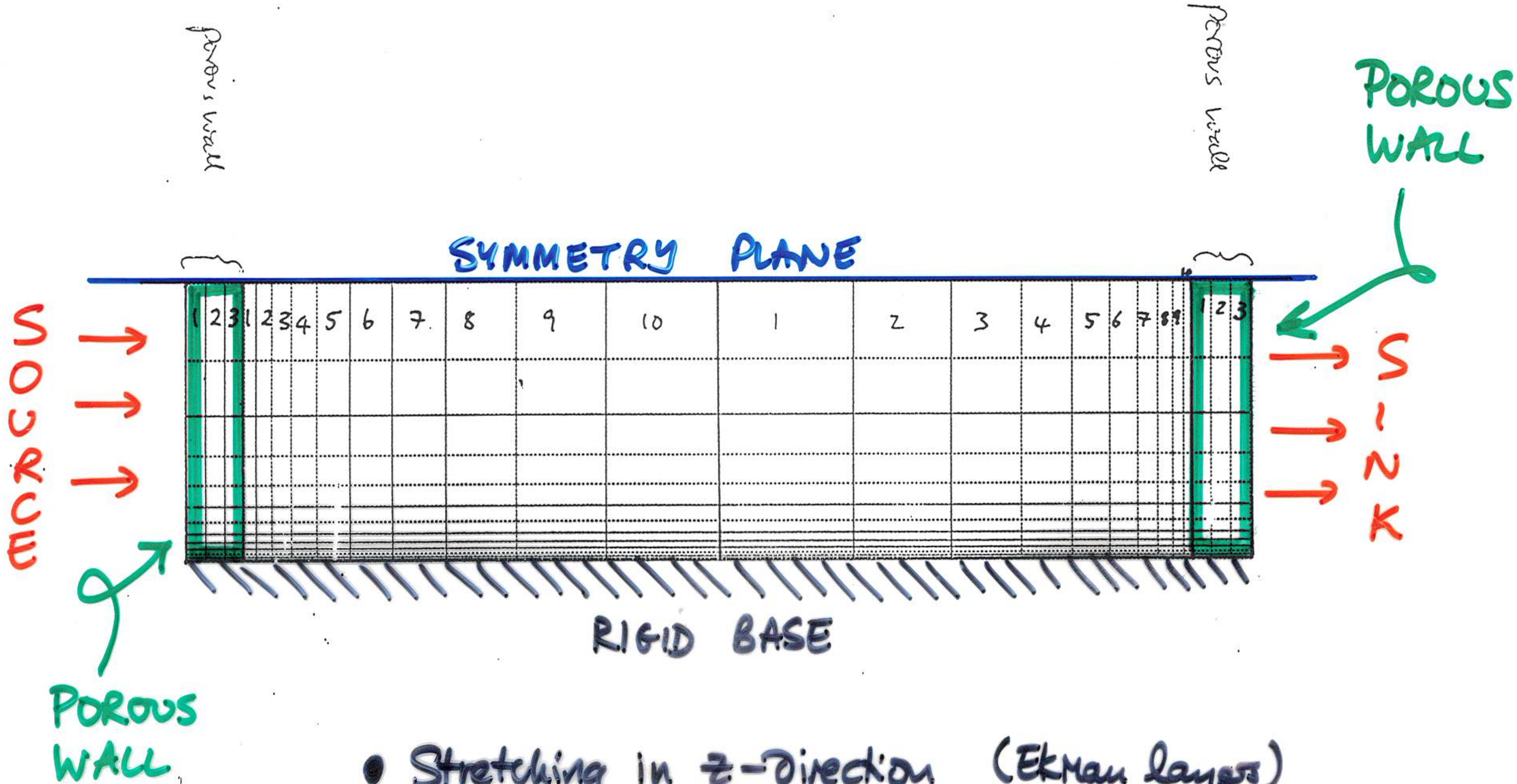
Newton-Raphson Solver.

- Uses analytic expressions for derivatives so:-

  $$\text{error}_{N+1} = (\text{error}_N)^2$$

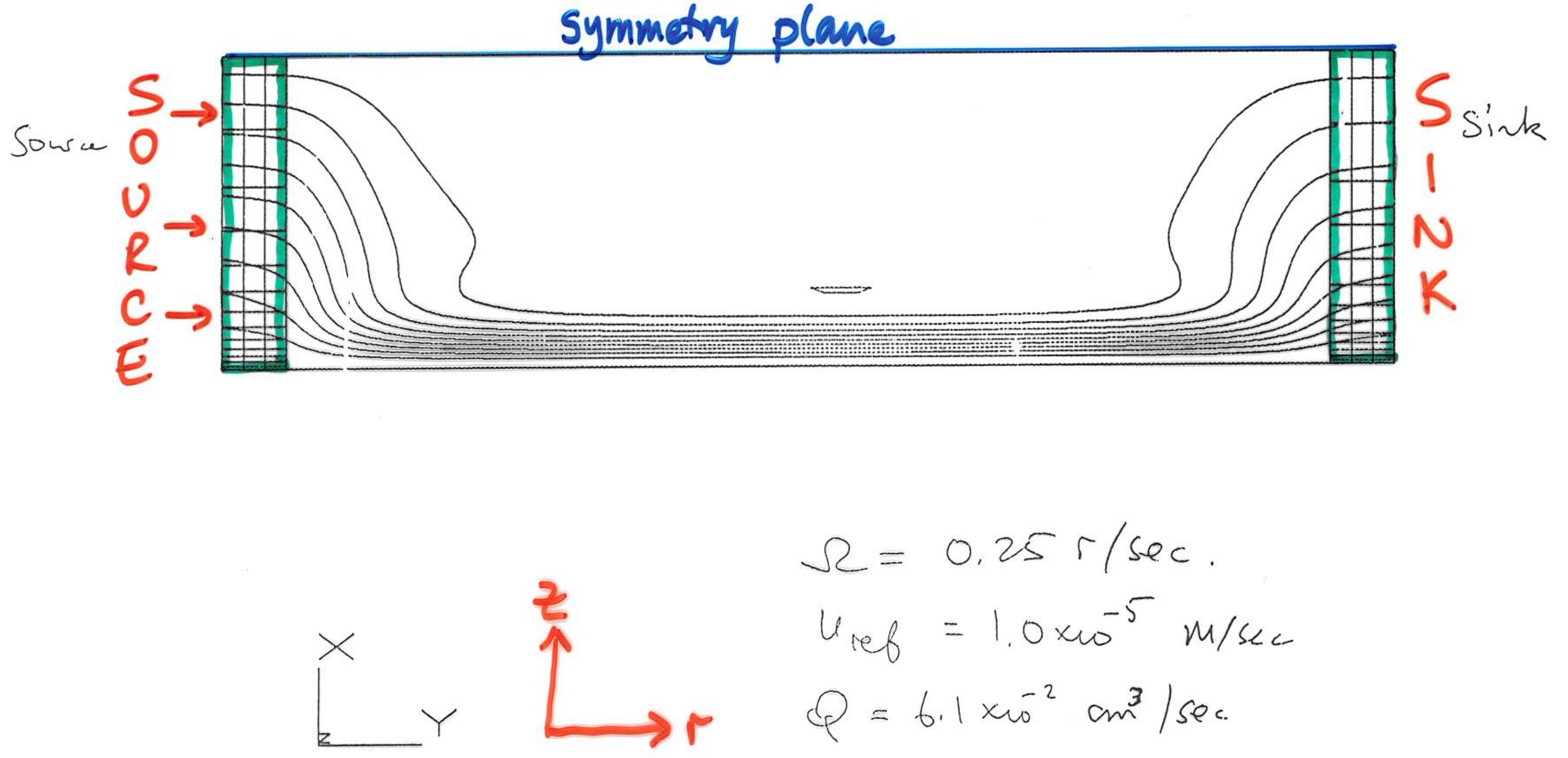
  Giving faster convergence than if derivatives were calculated numerically.
- Each iteration step matrix equation solved directly by Gaussian Elimination.

TYPICAL MESH.



- Stretching in z-direction (Ekman layers)
- Stretching towards source & sink

STREAMLINES



$\Omega = 0.25$ r/sec.

$u_{ref} = 1.0 \times 10^{-5}$ m/sec

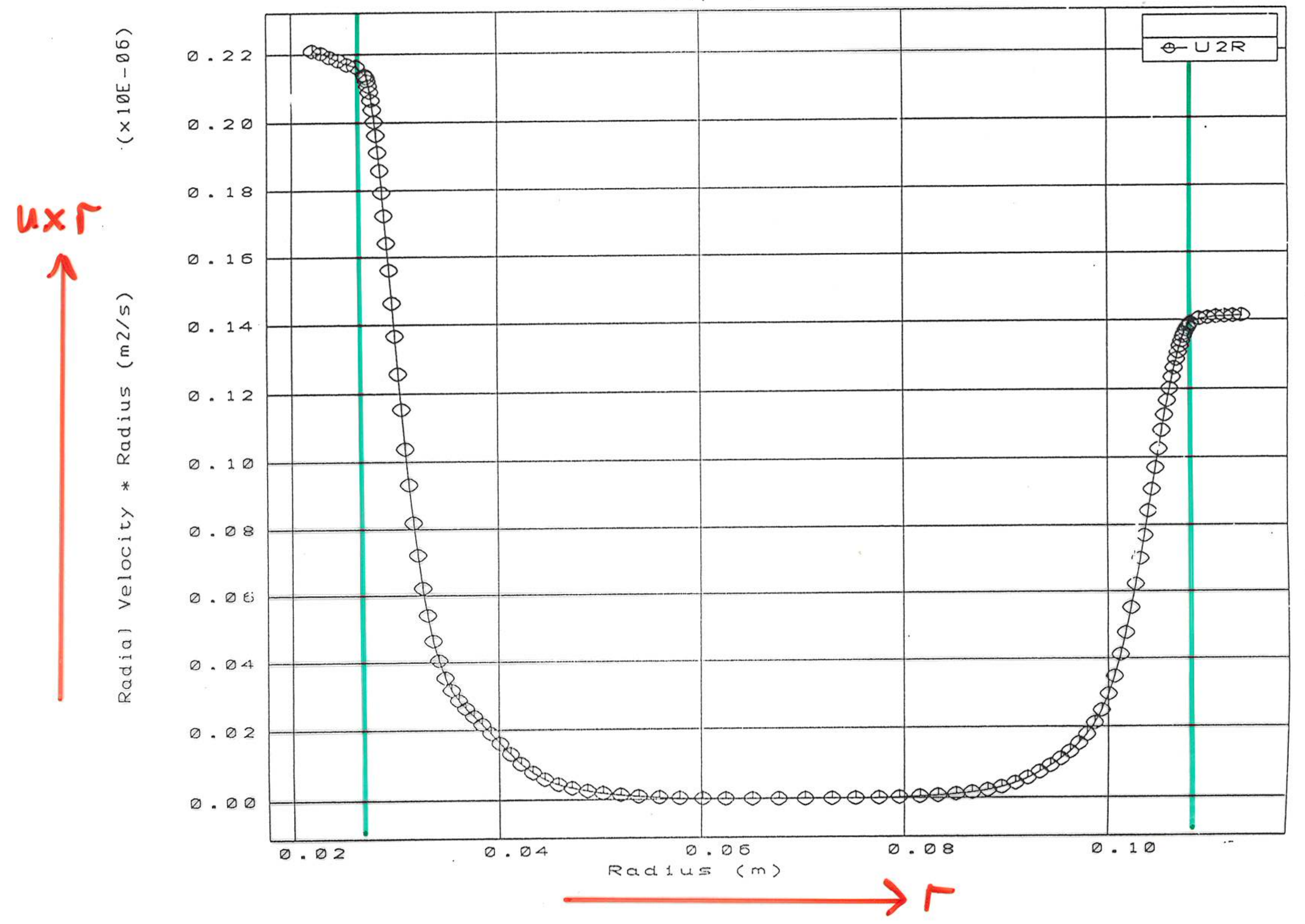
$Q = 6.1 \times 10^{-2}$ cm^3/sec.

Radial Velocity Component (x radius) - Symmetry plane

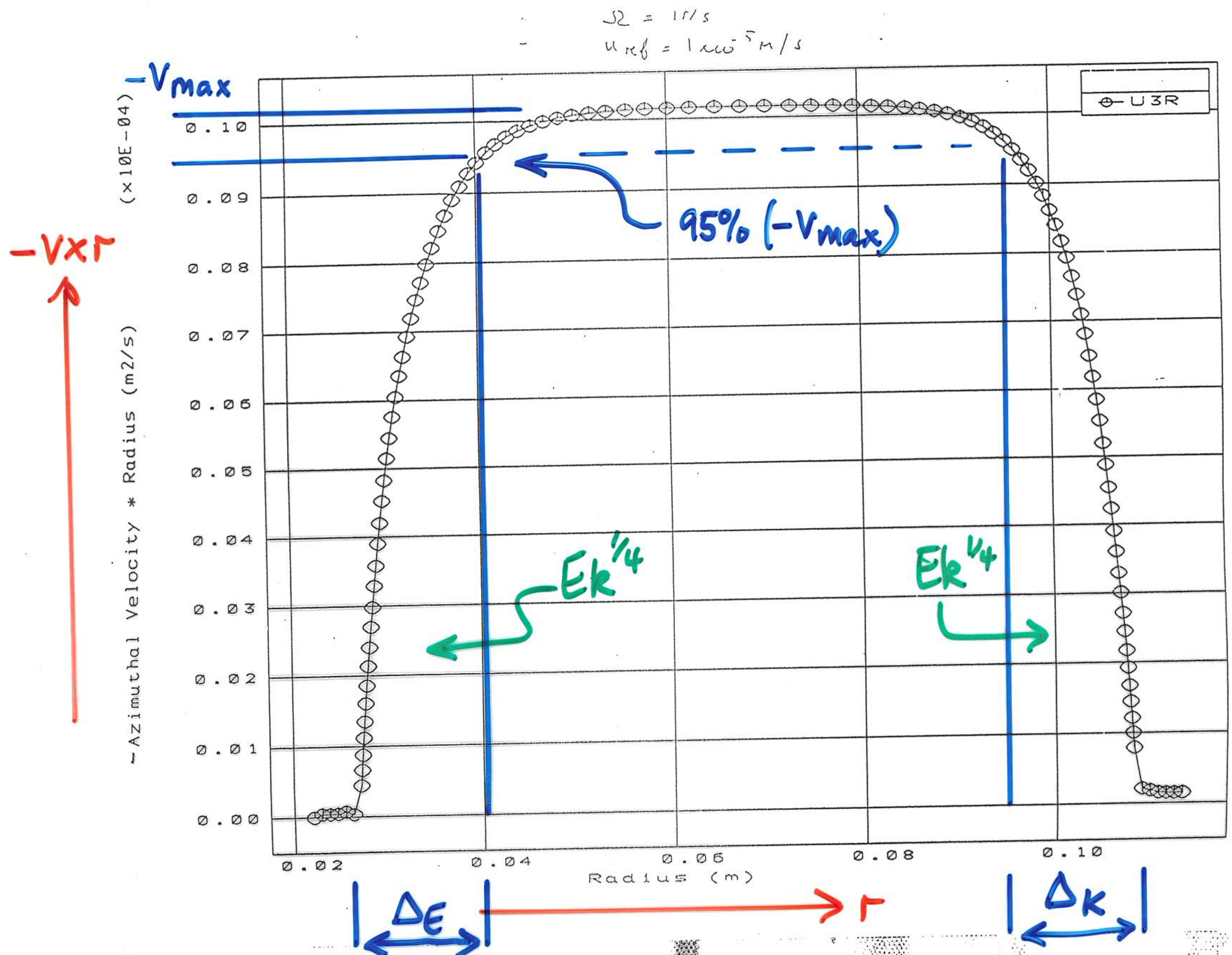
(Low Q)

$\Omega = 1$ r/s

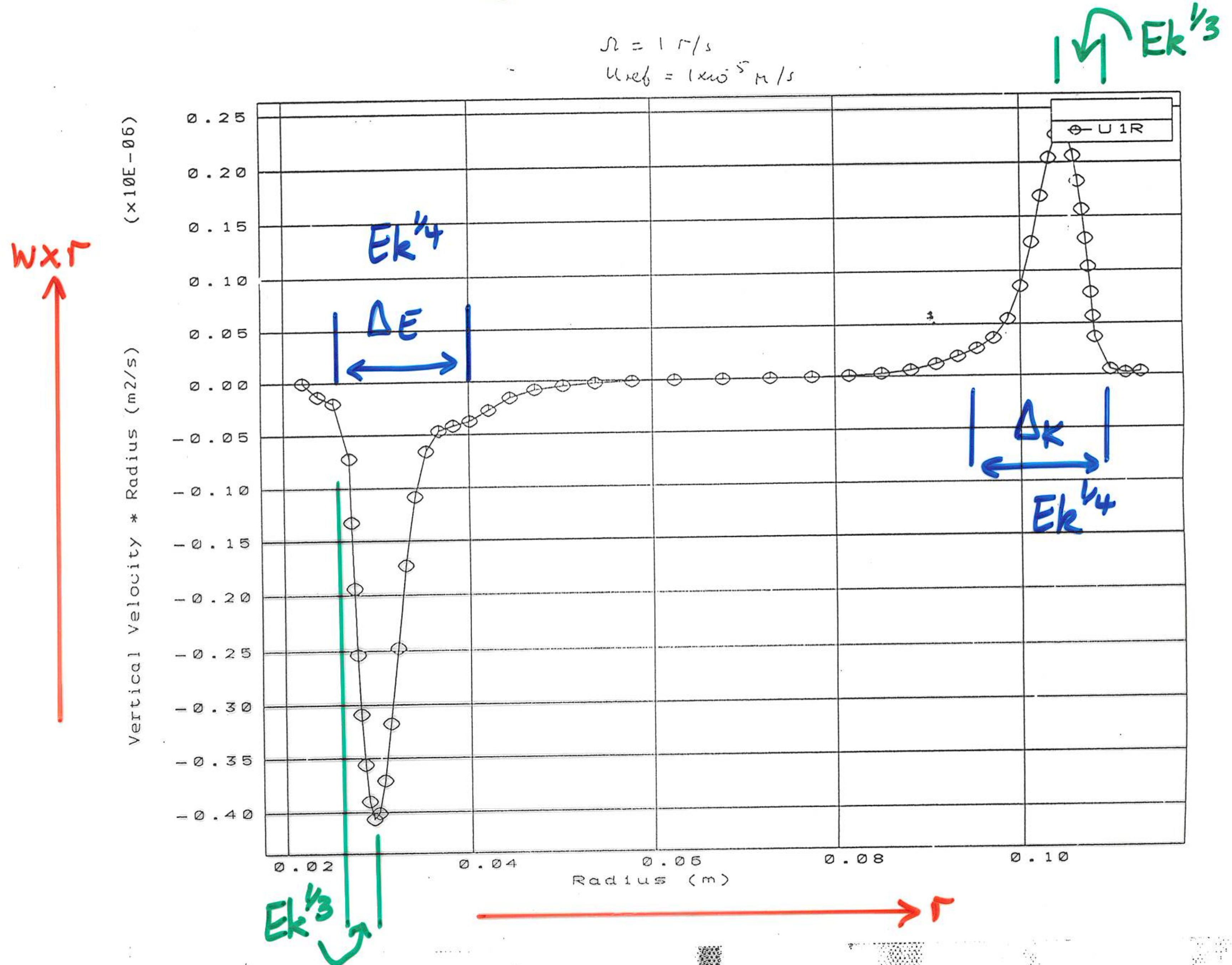
$U_{ref} = 1 \times 10^{-5}$ m/s



(-ve) Azimuthal Velocity (x radius) – symmetry plane (Low Q)



Vertical Velocity (x radius) – Quarter Height. (Low Q)



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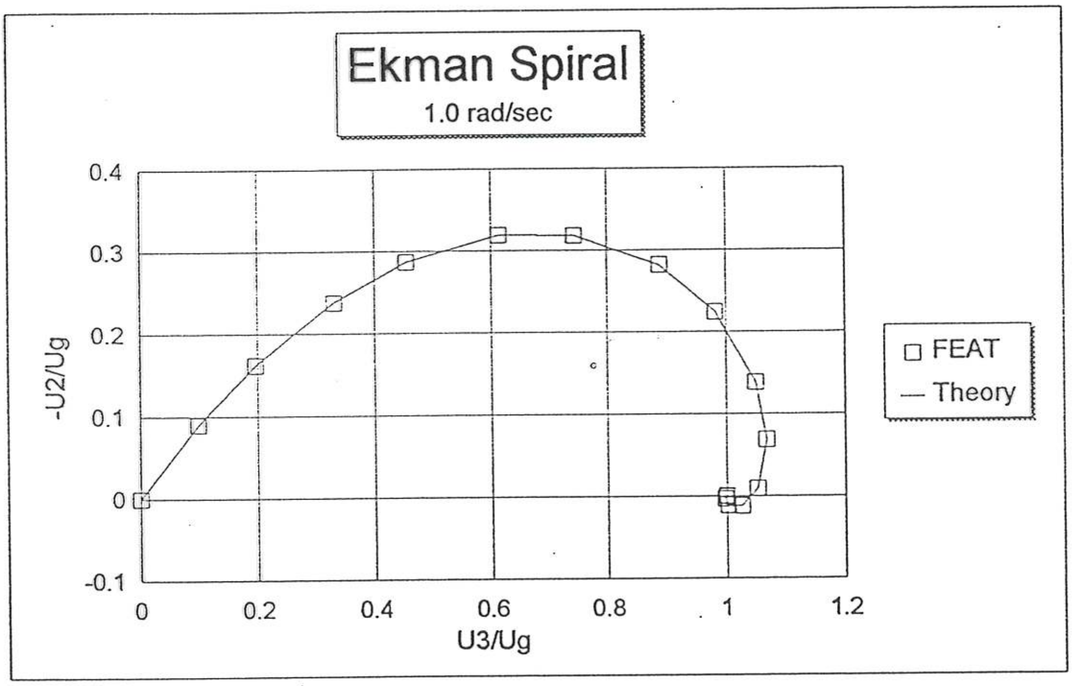
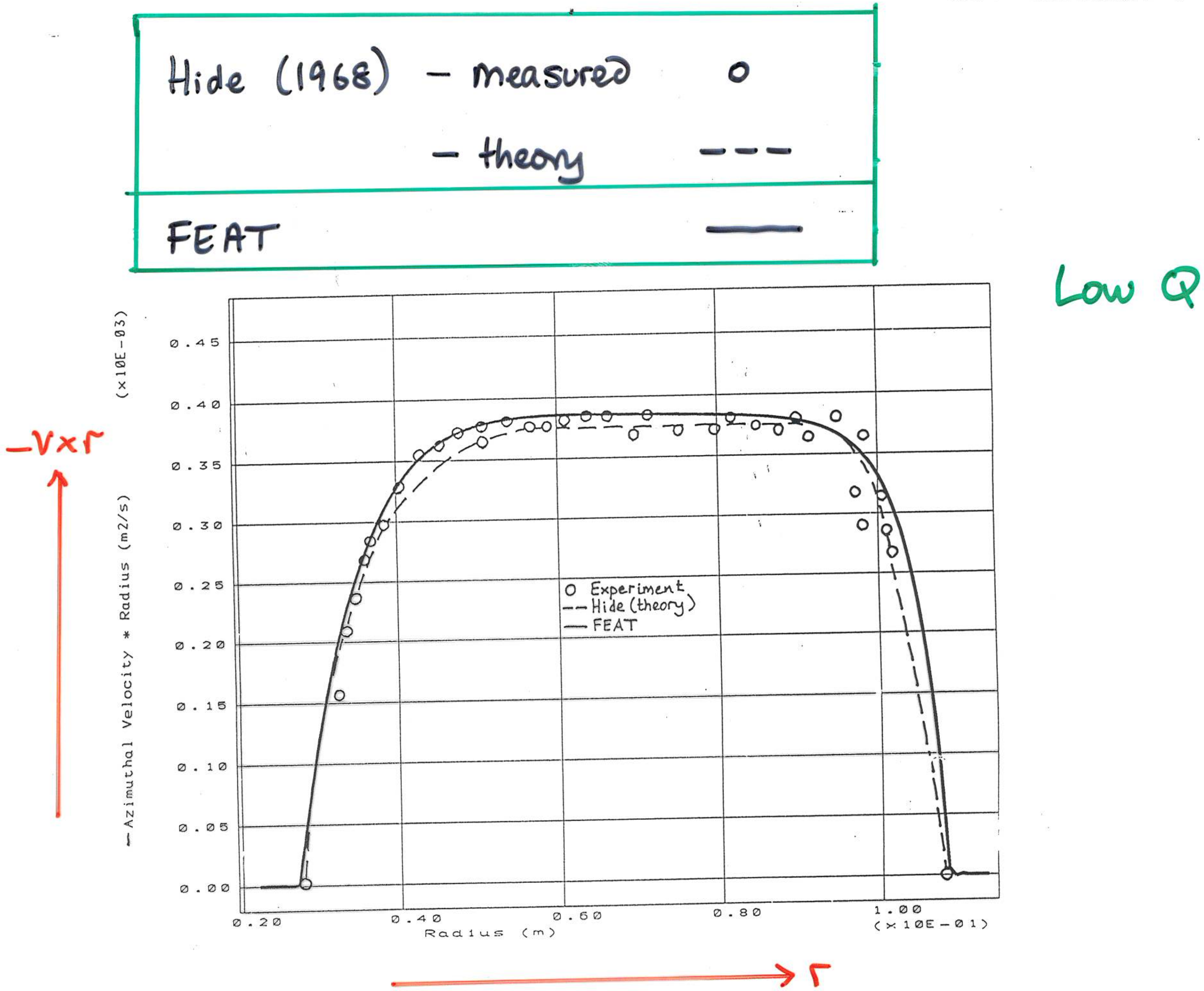
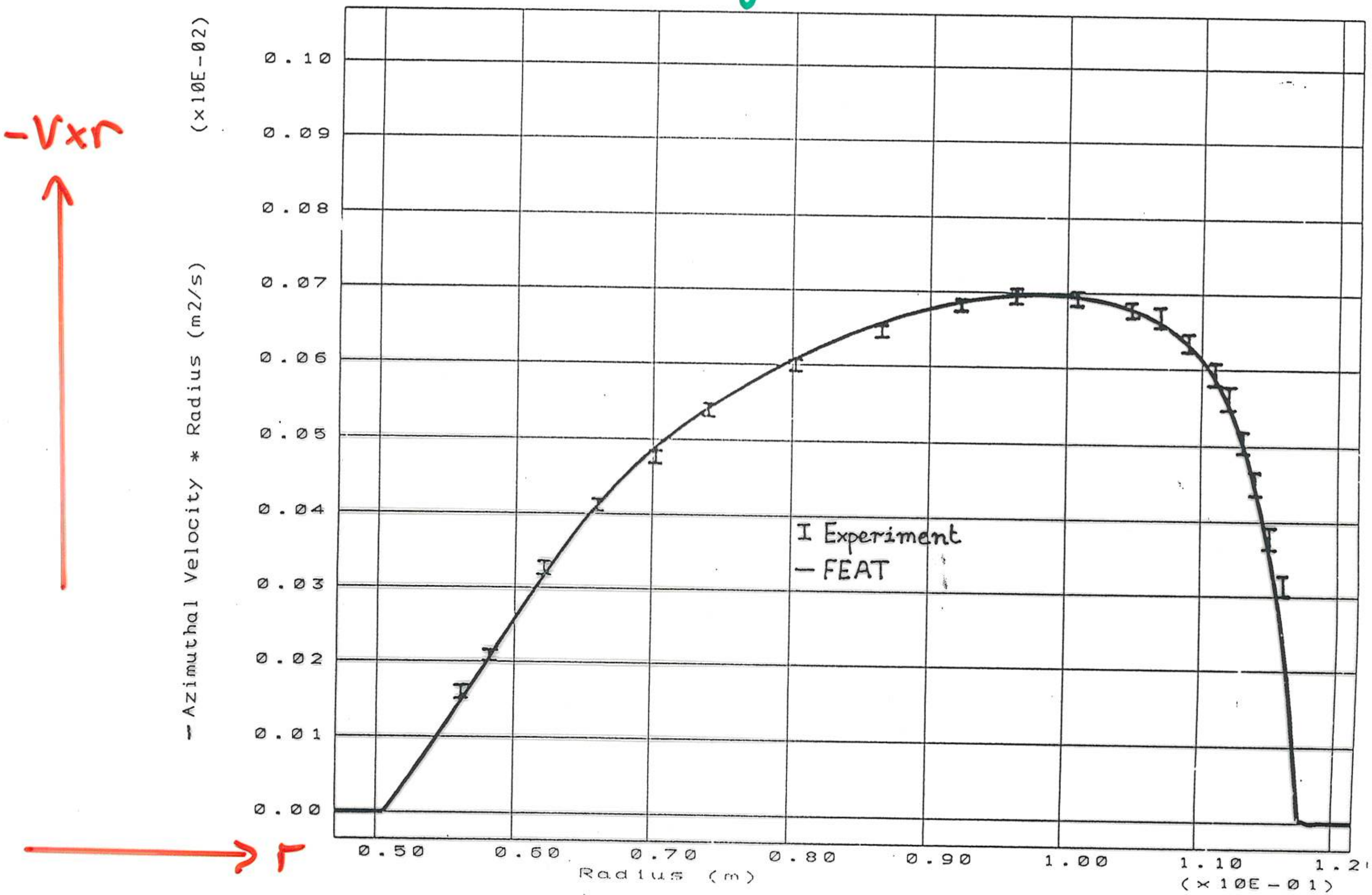


Ekman Spiral
1.0 rad/sec
0.4
0.3
0.2
0.1
0
-0.1
-U2/Ug
0
0.2
0.4
0.6
0.8
1
1.2
U3/Ug
FEAT
Theory


Hide (1968) – measured o
– theory - - -
FEAT ——
Low Q
– Azimuthal Velocity * Radius (m2/s)
(x10E-03)
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.20
0.40
0.60
0.80
1.00
(x10E-01)
Radius (m)
o Experiment
-- Hide (theory)
— FEAT
–V×r
r

Bennetts & Jackson (1974), measured	I
FEAT	—

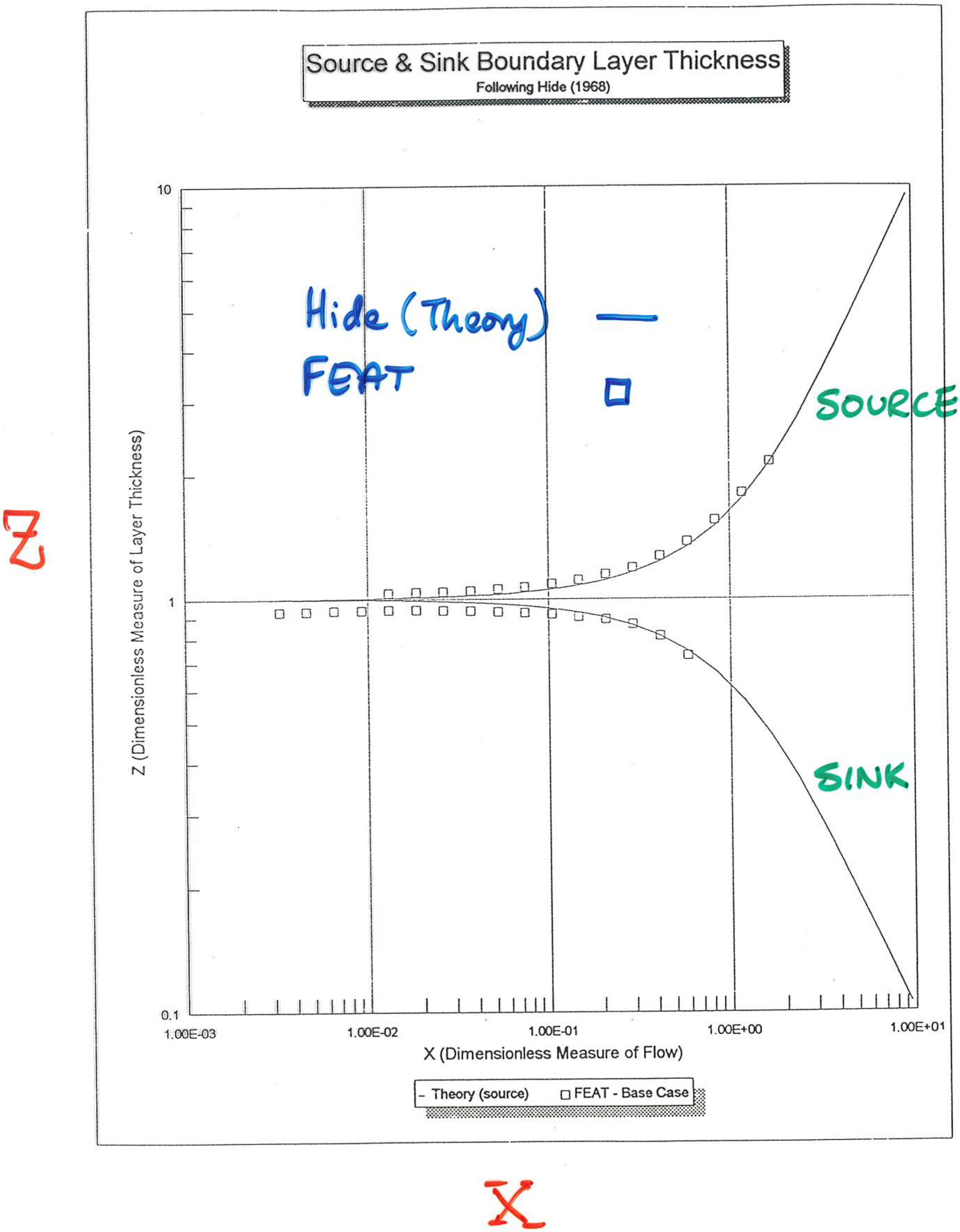
High Q



$Ro = 3 \times 10^{-1}$

$Ek = 8 \times 10^{-4}$

(22)



Source & Sink Boundary Layer Thickness
Following Hide (1968)
Hide (Theory) —
FEAT □
SOURCE
SINK
Z (Dimensionless Measure of Layer Thickness)
10
1
0.1
1.00E-03
1.00E-02
1.00E-01
1.00E+00
1.00E+01
X (Dimensionless Measure of Flow)
- Theory (source)
□ FEAT - Base Case
Z
X

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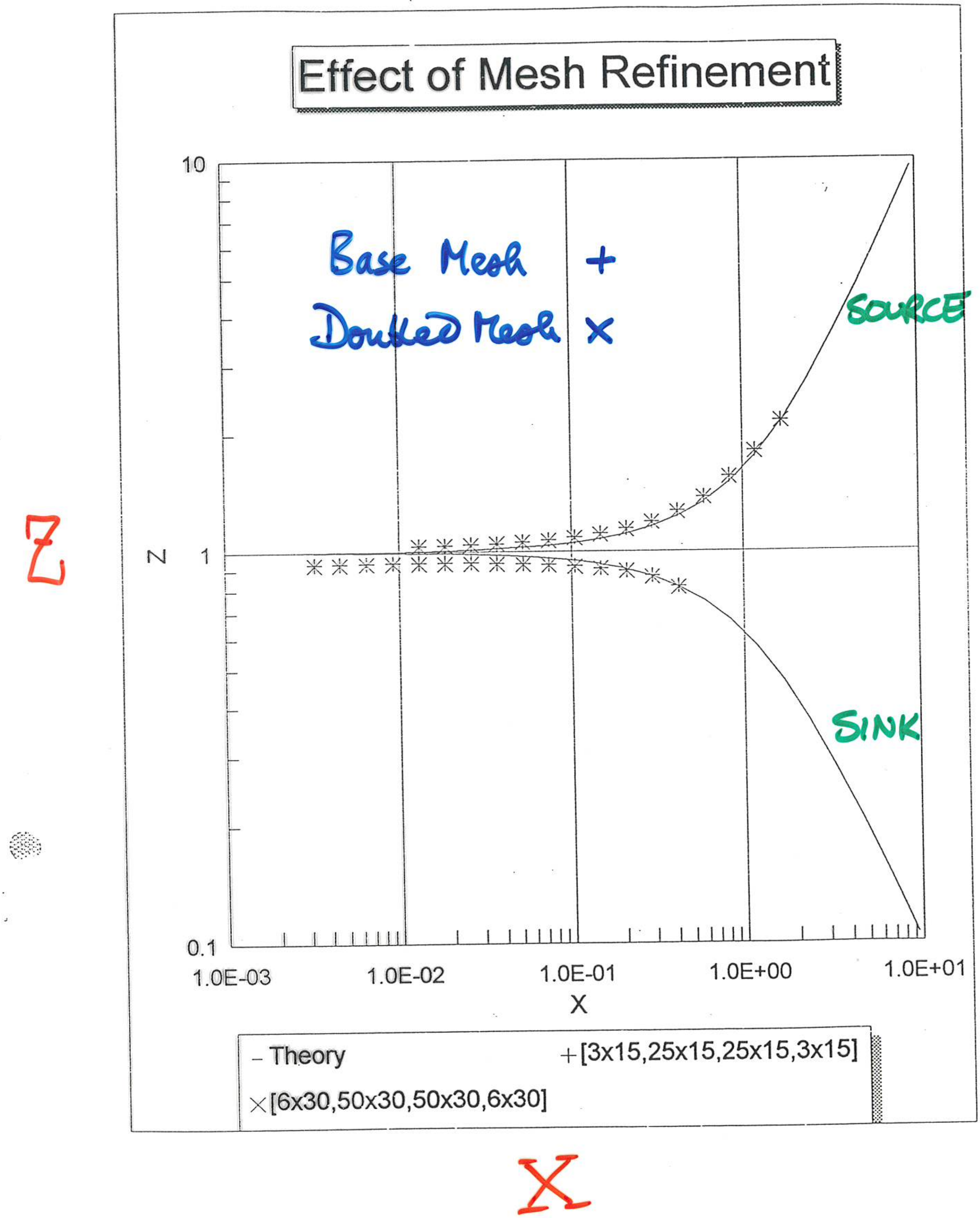
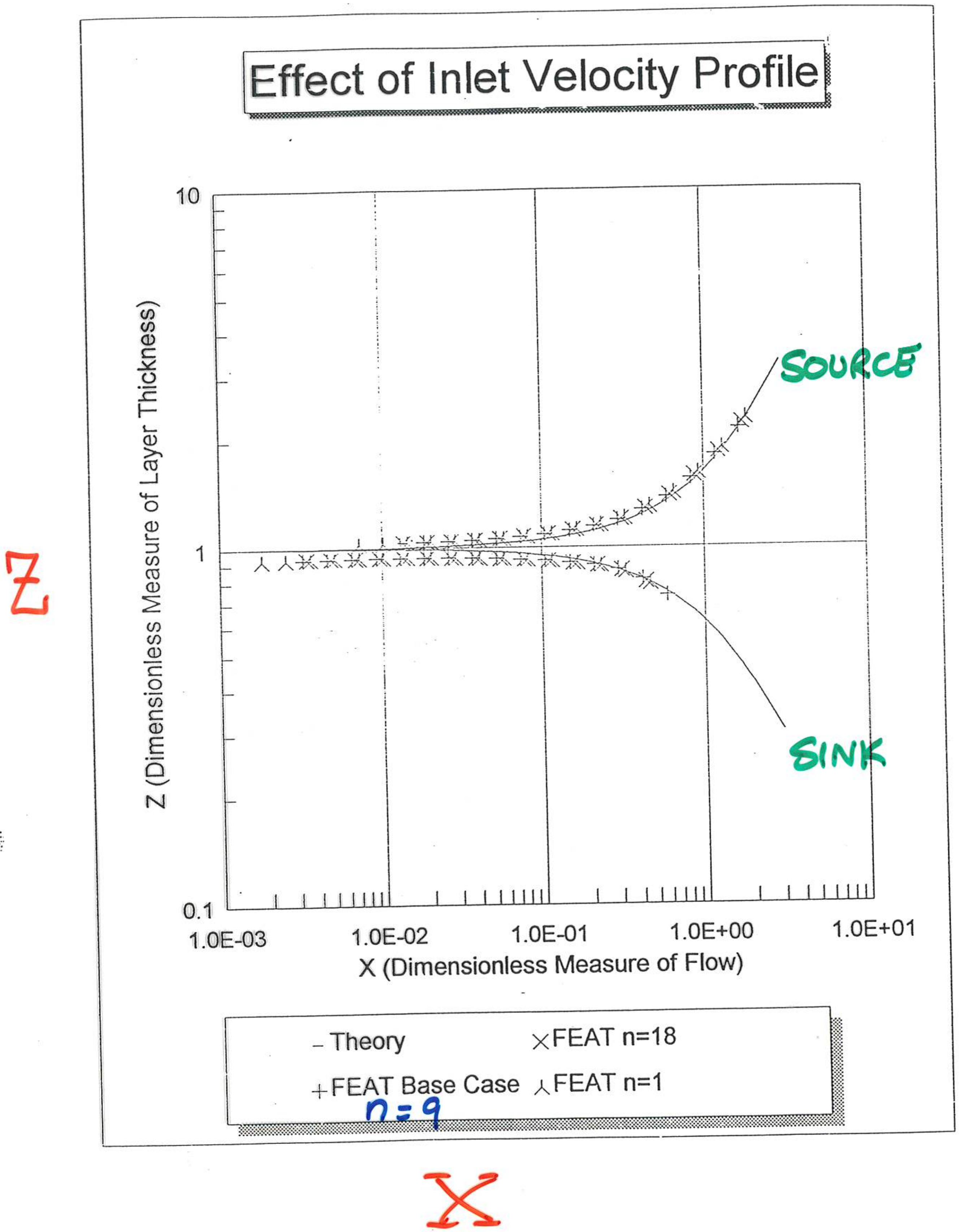

Effect of Mesh Refinement
Base Mesh +
Doubled Mesh ×
SOURCE
SINK
10
1
0.1
Z
1.0E-03
1.0E-02
1.0E-01
1.0E+00
1.0E+01
X
- Theory
+ [3x15,25x15,25x15,3x15]
× [6x30,50x30,50x30,6x30]
Z
X

(24)



$$u(z) = u_{ref}\left(1 + \frac{2z}{d}\right)^{1/n}$$

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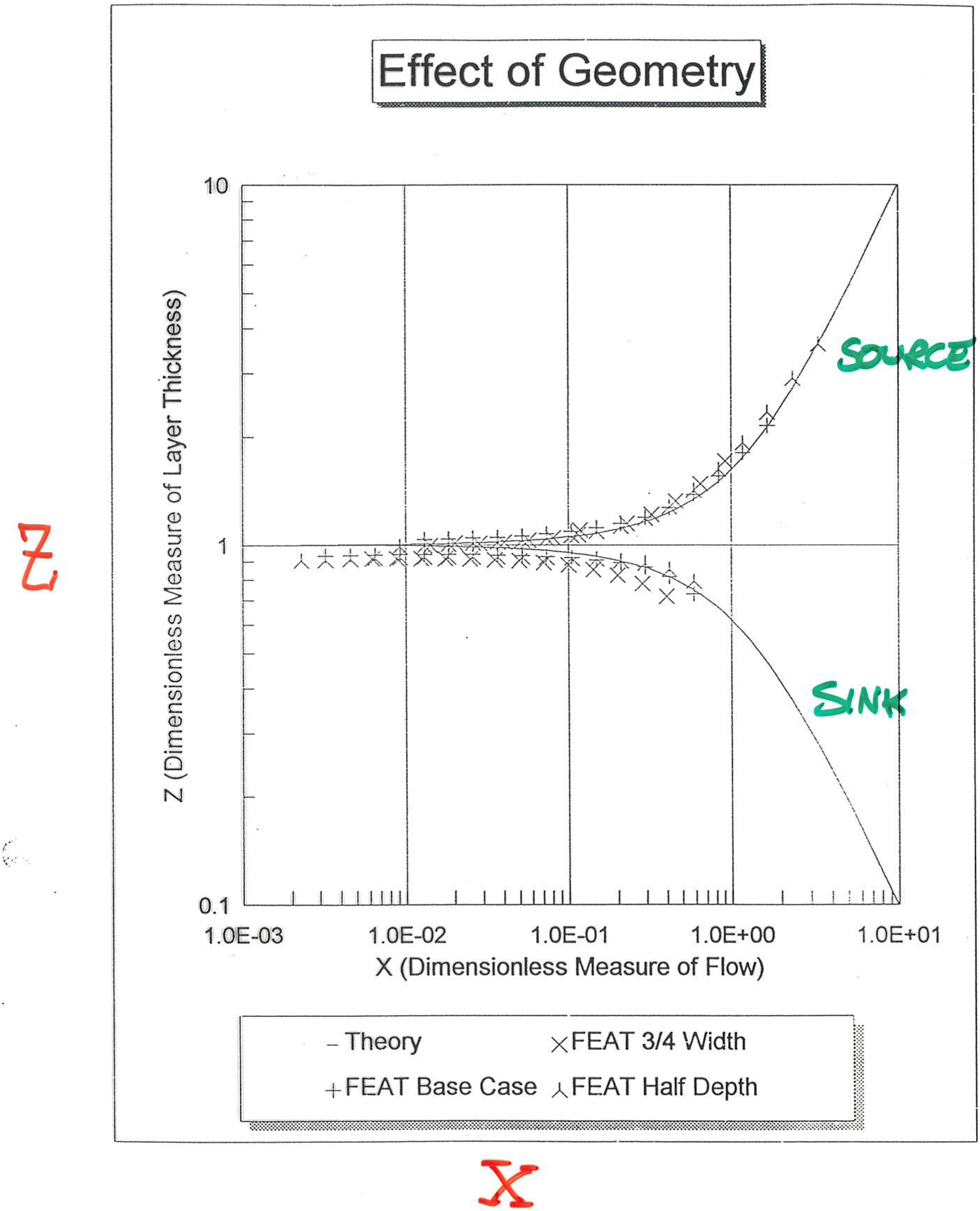
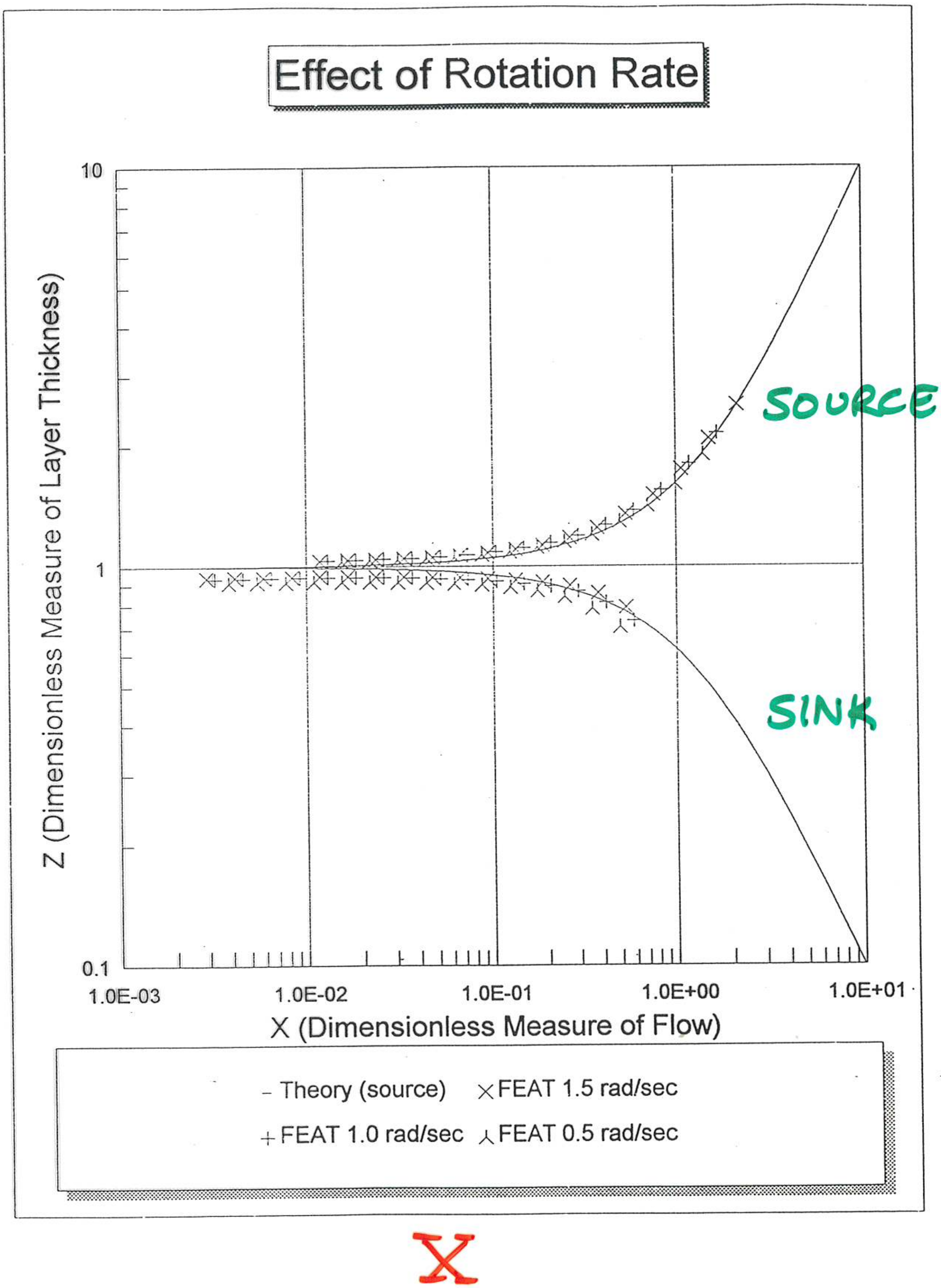
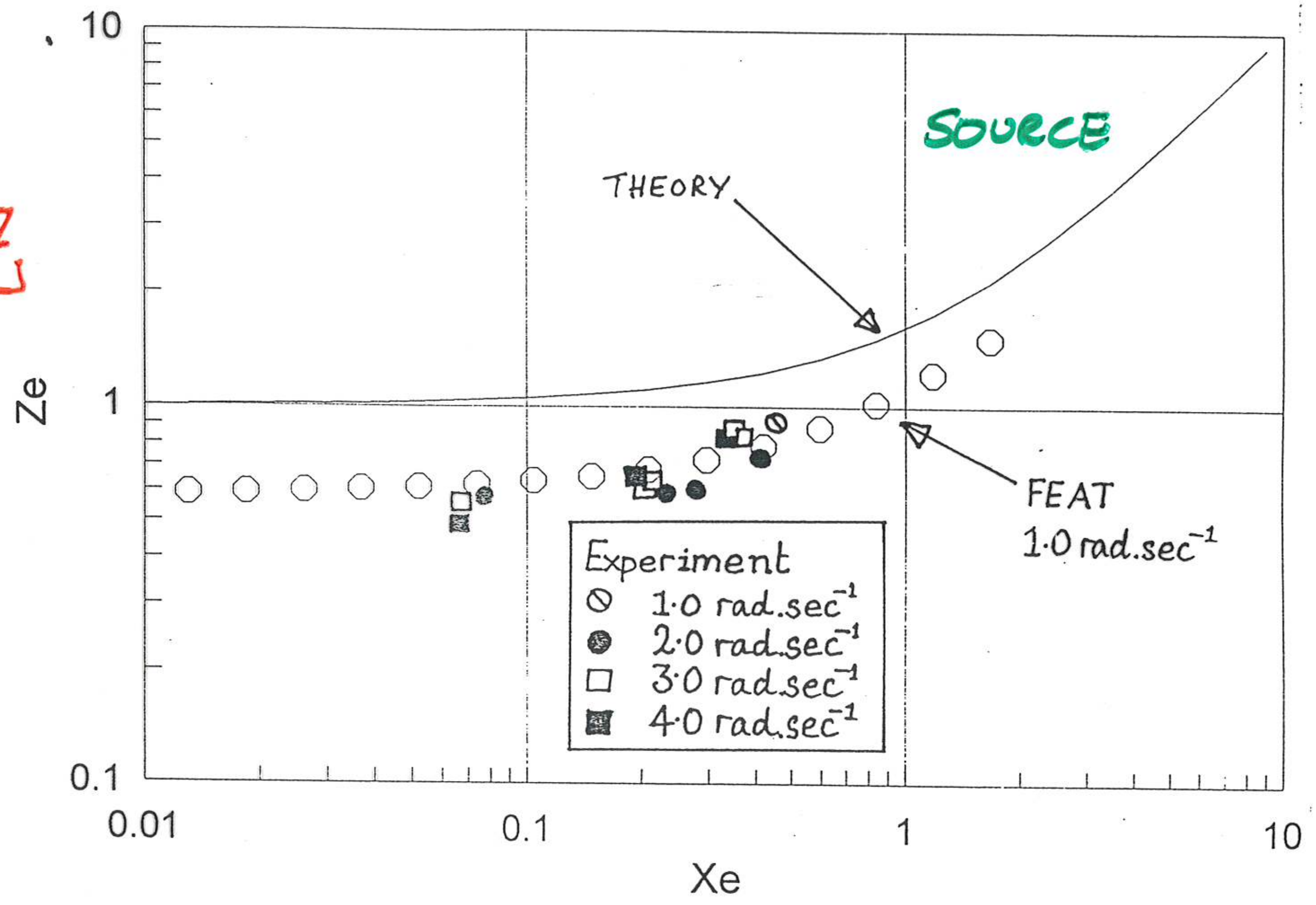

Effect of Geometry
Z (Dimensionless Measure of Layer Thickness)
10
1
0.1
1.0E-03
1.0E-02
1.0E-01
1.0E+00
1.0E+01
X (Dimensionless Measure of Flow)
SOURCE
SINK
Theory
FEAT 3/4 Width
FEAT Base Case
FEAT Half Depth
Z
X



Effect of Rotation Rate
10
1
0.1
Z (Dimensionless Measure of Layer Thickness)
1.0E-03
1.0E-02
1.0E-01
1.0E+00
1.0E+01
X (Dimensionless Measure of Flow)
Theory (source)
FEAT 1.5 rad/sec
FEAT 1.0 rad/sec
FEAT 0.5 rad/sec
SOURCE
SINK
Z
X

(27)

Z



X

FEAT 82% boundary-layer thickness.

Source Layer			Sink Layer		
X_E	Z_E		X_K	Z_K	
	Hide (1968)	FEAT		Hide (1968)	FEAT
1.304×10^{-2}	1.007	1.031	3.262×10^{-3}	0.998	0.926
1.844×10^{-2}	1.009	1.036	4.613×10^{-3}	0.998	0.929
2.607×10^{-2}	1.013	1.040	6.524×10^{-3}	0.997	0.931
3.687×10^{-2}	1.019	1.046	9.226×10^{-3}	0.995	0.932
5.214×10^{-2}	1.026	1.055	1.305×10^{-2}	0.993	0.933
7.374×10^{-2}	1.038	1.066	1.845×10^{-2}	0.991	0.933
1.043×10^{-1}	1.054	1.081	2.610×10^{-2}	0.987	0.932
1.475×10^{-1}	1.076	1.104	3.691×10^{-2}	0.982	0.930
2.086×10^{-1}	1.110	1.136	5.219×10^{-2}	0.974	0.927
2.950×10^{-1}	1.158	1.186	7.381×10^{-2}	0.964	0.922
4.172×10^{-1}	1.230	1.260	1.044×10^{-1}	0.949	0.915
5.899×10^{-1}	1.338	1.374	1.476×10^{-1}	0.929	0.905
8.343×10^{-1}	1.501	1.546	2.088×10^{-1}	0.901	0.890
1.180	1.751	1.798	2.952×10^{-1}	0.863	0.861
1.669	2.137	2.148	4.175×10^{-1}	0.813	0.808

FEAT Solution for Z Compared with Hide's (1968) Approximate Theory.

When source on inner cylinder

FEAT source layer slightly thicker than Hide's theory at low Q.

Sink layer slightly thinner.

Probably due to curvature (reverse flow direction, put source on outer cylinder to test)

SUMMARY.

- FEAT Validated against isothermal rotating fluid flow.
  - azimuthal flow profiles
  - Ekman b. layer
  - Source & Sink b. layers.
- Shown systematic agreement with theory for Δ_E & Δ_K over range of Ro, Ek considered.
- Shown that Hide's result is generally independent of:
  - inlet velocity profile
  - system depth & width changes
  - rotation rate

NEXT –

Reverse flow direction

Increase depth.