SIMULATION OF AXISYMMETRIC SOURCE-SINK FLOWS IN A ROTATING FLUID ANNULUS

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1 Thankyou Mr Chairman.

My name is Quintin Rayer, and my talk is titled Simulation of axisymmetric source-sink flows in a rotating fluid annulus.

I am currently working at Nuclear Electric Ltd in Gloucester.

2 The purpose of the work I am reporting here was two-fold.

Validation of the Nuclear Electric Ltd CFD code FEAT (Finite Element Analysis Toolbox) against rotating fluid flows and...

...to explore some interesting physics.

From my point of view there was the additional objective of learning some CFD!

For FEAT validation I looked for a very simple rotating fluid flow to simulate, hence

- axisymmetric flow, so 2D
- isothermal, so no buoyancy to worry about.

3 The rotating fluid annulus consists of fluid trapped between two co-axial cylinders which rest on a turn-table so that the axis of rotation coincides with the central axis of symmetry.

Fluid is pumped radially through the system from a porous source wall (the inner cylinder) out to the porous sink wall (the outer cylinder).

So the lid and base are rigid and impermeable, and the cylindrical side-walls are rigid but porous.

The whole system is then rotated uniformly about its central axis of symmetry - so the walls, lid and base all rotate together at the same angular velocity, Ω.

I have also indicated cylindrical polar coordinates, with radius, height and azimuth denoted by (r, ϕ, z).

3a (optional) So from above fluid is pumped into the annulus through the wall of the inner cylinder (the source) and exits through the porous wall of the outer cylinder (the sink).

3b Here is a section through the annulus in the (r, z)-plane.

The system is of depth d, but as the flow proves to be symmetrical about a plane at mid-height only the bottom half need be simulated.

Fluid enters through the inner cylindrical porous source wall at radius $r=a$ and exits at the sink wall at $r=b$.

The rotation vector is shown at the centre of the source.

The simulation also includes the porous walls at source and sink.

The tank dimensions are: inner radius, a=2.71cm, outer radius, b=10.83cm, depth, d=4.85cm. These were chosen to correspond with experimental work.

The rotation rate was 1.00rad/sec and the working fluid was water, so that the density was 10^3kg/m^3 and the kinematic viscosity 10^{-6}m^2/sec^{-1}.

So no-slip condition on the base, & vertical +azimuthal component of velocity zero in porous side-walls

6 (optional) The resulting flow, seen from above, feels the effect of the Coriolis acceleration...

...which turns it to the right, so that in the fluid interior flow is in the (-ve) azimuthal direction.

7 Here is a diagram of the resulting flow in the (r, z)-plane.

In the fluid interior the radial flow feels the effect of the Coriolis acceleration which turns it to the right (the -ve azimuthal direction).

This effectively prevents radial motion in the fluid body.

The only way fluid can traverse the chamber is by radial motion in the Ekman boundary-layers which form on surfaces perpendicular to the rotation axis.

Thus fluid entering the chamber at the source must move vertically down (only the bottom half of the chamber is shown here) in a source boundary-layer of thickness Δ_E (NB subscript E for sourcE)...

...to get into the Ekman boundary-layer so it can cross the annulus.

Finally it must rise in a region of thickness Δ_K (subscript K for sinK) to redistribute itself over the porous sink wall.

Please note the subscripts E for source and K for sink.

8 (optional) So how do I know about the resulting flow?

Experimental data give azimuthal flow profiles and esimates of source and sink boundary-layer thicknesses.

Theoretical studies give flow profiles and predications of Δ_E and Δ_K.

There has also been a single computational study (1974) which looked at a limited number of cases, checked some flow profiles and did a spot-check on the relative sizes of Δ_E and Δ_K.

9 To a large degree the interest resides in the side-wall boundary-layers.

Typically a set of simulations consists of setting the rotation rate and then gradually increasing the total volume flow rate through the annulus, Q.

The range of flow rates used were $Q=6\times10^{-2}$ to $8cm^3/sec$, giving inlet flow velocities of 10^{-3} - $10^{-1}cm/sec$.

The relevant dimensionless numbers here are the Rossby No, comparing inertial and rotational effects, and the Ekman No comparing viscous and rotational forces.

Ranges were $Ro = 2\times10^{-3} - 4\times10^{-1}$, $Ek = 3\times10^{-4} - 8\times10^{-4}$.

The Reynolds number was in the range 5×10^{-1} - 1×10^{2}, i.e. laminar flow.

The flow breaks down into two regimes:-

Linear regime, $Ro \ll Ek^{1/4} \ll 1$, the low flow case, where Δ_E, Δ_K equal the Stewartson boundary-layer thickness, a length-scale $\times$ $Ek^{1/4}$. In this case inertial effects can be neglected.

Non-linear regime, $1 \gg Ro \gg Ek^{1/4}$, the high flow case. Here the increased flow makes Δ_E thicken to Order Ro, while, at the sink, Δ_K is squashed to Order $Ek^{1/2}.Ro^{-1}$.

This also has the effects of skewing the azimuthal flow profile.

10 Graphically the results are something like this.

X is a dimensionless measure of the non-linearity and of the flow rate, Q. A Rossby number with a velocity-scale based on Q is combined with the Stewartson b-layer thickness made dimensionless by a suitable length-scale.

Z is the source (or sink) b-layer thickness divided by the

Stewartson layer thickness - so in the linear case (small Q, small X), Z→1.

X and Z are plotted on log scales.

Below two sketches illustrate how non-linear effects (high Q) cause the azimuthal flow profile to become skewed towards the sink.

The ~~flow~~ azimuthal flow profile is plotted as the (-ve) azimuthal velocity component × radius to remove the effects of curvature. It is plotted against radius across the annulus.

11 Here are some laboratory measurements of source-layer thickness plotted against his theory by Hide (1968), showing good agreement.

12 The FEAT simulation.

FEAT stands for Finite-Elements Analysis Toolbox.

It uses quadratic interpolation and a Newton-Raphson solver, making it implicit.

A mesh element is illustrated. All parameters apart from pressure have values recorded on corner and mid-face nodes (hence quadratic interpolation). The pressure need only be given on the nodes denoted L, but using a central node improves the representation of the continuity equation.

13 FEAT uses Galerkin Finite-Elements so there is no built-in upwinding, giving a centred-type scheme.

The Newton-Raphson solver uses analytic expressions for the derivatives of the equations, giving faster convergence than calculating the derivatives numerically.

At each iteration step the matrix equation is solved directly using Gaussian Elimination.

14 Here is an illustration of the mesh used.

Only the bottom half of the annulus is simulated, so there is a symmetry plane at the top, and a rigid impermeable base at the bottom.

The porous source and sink walls are also included.

The mesh is stretched in the vertical (z) direction to resolve the Ekman layers, and stretched towards the source and sink walls to resolve the side-wall layers.

The stretching method keeps a constant ratio between sucessive mesh element widths.

15 Here's a set of streamlines from a FEAT simulation.

You can see that the flow travels down in a layer by the source, moves radially across the annulus in a layer at the bottom, before rising in another layer region to exit at the sink.

16 (optional) This is a graph of the radial component of the fluid velocity × radius plotted against radius along the symmetry plane (i.e. mid-depth in the annulus).

The radial velocity is non-zero in the source and sink boundary-layer regions, but zero in the fluid interior, as there all the radial flow is in the Ekman boundary-layers.

17 This is a graph of the (-ve) azimuthal component of velocity × radius plotted against radius along the symmetry plane (i.e. mid-depth in the annulus).

Ro $\backsim 10^{-3}$, Ek $\backsim 10^{-4}$.

The azimuthal velocity increases from zero at the side-walls to its full value in the fluid interior in regions of thickness Δ_E and Δ_K. As this is a linear (low Q) case these layers are of thickness Order $Ek^{1/4}$ (Stewartson layers).

Also shown is the method used to determine the 95% boundary-layer thicknesses used in layer graphs of results. The maximum magnitude from the plateau of -Vr is taken, and 95% of this read off the graph (using linear interpolation between points on the graph if need be) to give the side-wall layer thickness. (In practice FEAT macro coding was set up to do this automatically.)

18 For the same case this is the corresponding vertical velocity profile (w×r vs radius). In this case it has to be taken at a height 1/2-way up the simulated region, 1/4-way up the annulus (at mid-depth w=0, by symmetry).

It shows fluid descending in the source layers and ascending in the sink layers.

It also reveals the $Ek^{1/3}$ sub-structure in the Stewartson layers, where the vertical velocity is reduced from its peak value to zero at the side-walls.

19 (optional) As a check that FEAT is correctly reproducing the Ekman b-layers, the simulation results have been compared with the theory for the Ekman spiral, giving excellent agreement.

20 It is not too hard to measure the azimuthal velocity profile in the laboratory. These results compare Hide's measured data with his theoretical flow profile and FEAT simulation results.

This is pretty much in the linear (low Q) regime, and there is excellent agreement between FEAT and the measurements.

21 A similar azimuthal flow profile was measured by Bennetts & Jackson in the non-linear (high Q) regime, where the profile is considerably skewed over towards the sink.

The results are compared with FEAT and show excellent agreement.

22 As I mentioned before FEAT can also be used to derive the source and sink b-layer thicknesses, which can then be plotted against Hide's theory.

Here is a graph of X (measure of Q and degree of non-linearity) and Z (side-wall b-layer thickness).

The line indicates Hide's theory, calculated using an assumed flow profile, compared with the FEAT layer thicknesses (squares).

There is excellent agreement.

You can see that at small X, whereas Hide's results assymptote to 1, the FEAT sink layer tends to a value smaller than the FEAT source layer.

This may be a curvature effect, which might be changed by putting the source on the outer cylinder and and sink on the inner cylinder.

The results have been checked by doubling the number of mesh elements in both directions - which has negligible effect on the simulation solution. (Slide 23 optional.)

One of the difficulties of comparing a simulation with experimental data is knowing certain details of the laboratory flows. For example the precise source inlet velocity profile as a function of height.

To examine this sensitivity studies were carried out in the simulation using extremes from a linear inlet flow profile upto an 1/18th power law (very flat). In most cases a 1/9th power law was used.

The precise flow profile used (even the linear one) had negligible effect on the results obtained. (Slide 24 optional.)

Other tests related more to the physics of the problem.

Simulations where the depth of the annulus was halved, or the width reduced to 3/4 of its initial value had very little effect on the side-wall layer thicknesses as expressed by Z and X.

In fact reducing the width had a small effect - which is consistent with the idea that the curvature may account for the differences between the FEAT solution and Hide's theory at low X. (Slide 25 optional.)

Another test involved changing the rotation rates from the base case value of 1.0 rad/sec to 1.5 and 0.5 rad/sec. Again the results were closely in agreement with Hide's theory. (Slide 26 optional.)

27 (optional) Hide's experimental determinations of source-layer thickness, and their slight discrepancy from the theoretical values can be reproduced by making a systematic variation to the side-wall b-layer thickness used.

In this case FEAT shows excellent agreement with the measurements by using an 82% b-layer thickness.

28 (optional) Hide's theory is compared with FEAT in tabular form, revealing the slight discrepancy in Z for small X.

The FEAT results are for a source on the <u>inner</u> cylinder.

The difference is probably due to the effects of curvature.

29 Summary.

FEAT has been validated against an isothermal rotating fluid flow.

There has been good agreement with:-

- azimuthal flow profiles
- Ekman boundary layers (Ekman spiral)
- source and sink boundary layers.

FEAT has shown systematic agreement with the theory for the source and sink b-layers over the range of Ro and Ek covered.

The results also support Hide's theory for the side-wall layer thicknesses, and that it is quite robust to:-

- inlet velocity profile changes
- system geometry (depth and width changes)
- rotation rate

The remaining work which needs to be carried out is:-

- reverse the flow direction (find out what the effect on Z really is)

- Increase the annulus depth (make it harder for the fluid to get through)

Thank-you for listening, any questions?