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SIMULATION OF THE FLOW IN A DIFFERENTIALLY HEATED ROTATING FLUID ANNULUS

Q.G.RAYER,
Nuclear Electric Plc., Berkeley Technology Centre, Berkeley, Glos. GL13 9PB, UK.

B.G.SHERLOCK,
Department of Electrical Engineering, Parks College of Saint Louis University, 500 Falling Springs Road, Cahokia, Illinois, 62206, USA.

S.AHMED,
Department of Computer Science, Parks College of Saint Louis University, 500 Falling Springs Road, Cahokia, Illinois, 62206, USA.

Abstract A control-volume approach is used to simulate the incompressible flow in a differentially heated uniformly rotating fluid annulus. Pressure and velocities are calculated iteratively at each time-level, yielding a semi-implicit scheme. Buoyancy effects are included by using the Boussinesq approximation. The model is tested against observations of the flow in an annulus with a radial barrier. Differential heating is provided by maintaining the side-walls of the annulus at separate temperatures.

INTRODUCTION

Geophysical systems (e.g. atmospheres and oceans) show circulations caused by differential heating and rotation. Their understanding is assisted by the study of flows in a differentially heated rotating fluid annulus (Figure 1), which shares rotational and thermal forcing, and has simple, well defined boundary conditions. In the annulus fluid is held between co-axial cylinders and rigid thermally insulating end walls, and rotates with uniform angular velocity Ω about the central axis of symmetry. Thermal forcing is provided by holding the side walls at different uniform temperatures, T_a and T_b.

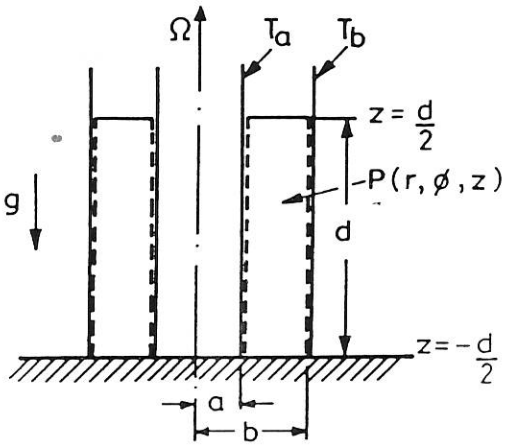


FIGURE 1. Diagram of fluid annulus. (r,ϕ,z) are cylindrical polar coordinates of a point P, in a frame rotating with the annulus at Ω rad.s^{-1}. $\Delta T \equiv T_b - T_a = 4\,°C$, a=2.5cm, b=8.0cm, d=14cm.

Previous experiments have shown that the annulus fluid heat transport decreases as Ω increases[1]. However if it is blocked by a thin radial barrier the heat advection is almost independent of Ω[2,3]. The causes have been investigated experimentally[2] and by a computer model[2,4].

The model previously used[4] was a grid-point finite-difference scheme based on the Navier-Stokes equations for incompressible flow in a Boussinesq fluid, using a stretched grid in the (r,z) plane to resolve boundary layers[5-7].

This model failed to accurately simulate several aspects of the flow with a barrier. Defects in the velocity field have been noticed[2,7], and it inaccurately calculated the temperature drop observed across the thermally insulating barrier[2]. Another likely weakness was the absence of sufficient grid points to represent any barrier boundary layer.

These difficulties are being addressed by a new code which uses a control-volume scheme[8,9]. It is currently being validated against measurements before being used on problems which lie beyond current experimental techniques.

CONTROL-VOLUME COMPUTER MODEL

The computer model uses a control-volume approach[8,9]. The discretization equation thus obtained expresses global conservation principles for quantities such as mass, momentum and energy which are exactly conserved over the calculation domain. The dynamical equations used can be expressed in non-dimensional form as:

$$\frac{\partial \underline{u}}{\partial t}+\nabla.(\underline{u}.\underline{u})=-\nabla p+\mathrm{Gr}(\mathrm{Ro})^2T\hat{z}-r\alpha T\hat{r}-2u\hat{\phi}+2v\hat{r}+\mathrm{Ro}\nabla^2\underline{u},$$

$$\nabla.\underline{u}=0, \text{ and } \frac{\partial T}{\partial t}+\underline{u}.\nabla T=\frac{\mathrm{Ro}}{\mathrm{Pr}}\nabla.(k\nabla T).$$

The non-dimensional parameters are fluid velocity, $\underline{u}$ = (u,v,w); time, t; pressure, p; temperature, T; thermal expansion coefficient, α; and thermal conductivity, k. Gr, Pr and Ro are suitable representations of the Grashoff, Prandtl and Rossby numbers respectively.

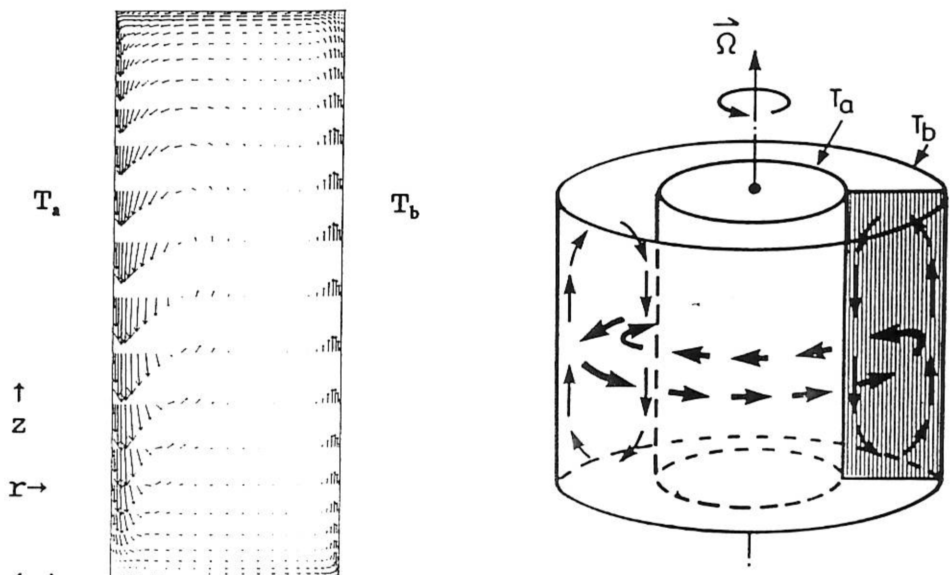
MODEL RESULTS

A simulation with ΔT=4°C and Ω=0.5rad.s^{-1} is given in Figure 2. Measurements are qualitatively reproduced in the (r,z) plane, but not as well in the (r,ϕ) plane (not shown). The spurious eddy motion mentioned by Rayer[2] is not seen.

Experiments show an azimuthal temperature gradient and a temperature drop across the barrier, ΔT_B, which is reproduced. The sense of ΔT_B is correct, but the model gives ΔT_B=0.8°C compared with a measurement of about 0.2°C.

Thus the code replicates aspects of the

measured flow, but further development is required.



(a) (b)

FIGURE 2. (a) Model results. (b) Schematic of experimental flows.

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