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Title: Numerical simulation of flow in a differentially heated rotating fluid.

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A new approach is taken to solve a three-dimensional incompressible fluid flow problem with differential heating in a uniformly rotating frame. The Navier-Stokes equation for a rotating frame is discretised using a finite volume approach which ensures that global conservation laws are obeyed. The semi-implicit scheme is formally first-order accurate in time and second-order in space. Buoyancy effects, due to temperature gradients in the fluid, are included by using the Boussinesq approximation,

$$\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}.\nabla)\,\boldsymbol{u}+2\boldsymbol{\Omega}\times\boldsymbol{u}=-\frac{1}{\rho_0}\nabla p+\nabla\Phi-\alpha\,(T-T_0)\,\nabla\Phi+\nu\nabla^2\boldsymbol{u}.$$

where **u** is the velocity of a fluid element, Ω is the uniform angular velocity of the rotating frame, ρ_0 is the mean density of the fluid, p is the pressure, ν the (assumed constant) kinematic viscosity of the fluid and Φ the potential of external forces. In cylindrical polar coordinates (r,ϕ,z), $\nabla\Phi = \mathbf{g} - \Omega \times (\Omega \times \mathbf{r})$, where $\mathbf{g} = -g\mathbf{k}$ is the acceleration due to gravity. By considering only incompressible flow, the equation of conservation of mass becomes $\nabla.\mathbf{u} = 0$. If the fluid density is linearly dependent on temperature T, the density of the incompressible fluid is given by

$$\rho=\rho_0\,[1-\alpha\,(T-T_0)\,]\,,$$

so that ρ_0 is the density of the fluid at the mean fluid temperature T_0 and α is the coefficient of thermal expansion for the fluid. The form of the energy equation

$$\frac{\partial T}{\partial t}+\boldsymbol{u}.\nabla T=\kappa\nabla^2 T,$$

neglects the viscous dissipation of energy, the presence of internal energy sources and sinks, and work done by the body force. For the application envisaged, this is an excellent approximation, however it would be simple to incorporate these effects into the scheme if desired.

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The performance of the model will be tested against experimental measurements of the incompressible viscous flow in a differentially heated fluid annulus rotating with uniform angular velocity, including the effects of non-uniform bases and the introduction of radial barriers. Differential heating is provided by maintaining the side-walls of the annular convection chamber at separate temperatures. The range of experimental parameters to be modelled are given in Table 1.

Radius of inner cylinder	a	2.5 cm
Radius of outer cylinder	b	8.0 cm
Mean depth of convection chamber	d_0	10.6 - 14.0 cm
Angular velocity	Ω	0.0 - 5.0 $rad.sec^{-1}$
Mean fluid temperature	T_0	20 °C
Applied temperature difference	ΔT	4 or 10 °C
Kinematic viscosity of fluid	ν	$1.8x10^{-2}$ $cm^2.sec^{-1}$
Specific heat capacity of fluid	C_P	3.84 - 3.85 $J.g^{-1}.°C^{-1}$
Mean density of fluid	ρ_0	1.05 - 1.09 $g.cm^{-3}$
Expansion coefficient of fluid	α	$3.03x10^{-4}$ $°C^{-1}$
Thermal conductivity of fluid	k	$5.2x10^{-3}$ $W.cm^{-1}.°C^{-1}$

Table 1: Range of laboratory parameters.

The range of experimental configurations used for testing the model will require considerable flexibility in the model grid geometries. Studies of the rotating annulus have previously contributed to the understanding of global atmospheric and oceanic flows. In this case the model is to be used to investigate the properties of a quantity known as the superhelicity density of the fluid,

$$\mathcal{S} \equiv \boldsymbol{\omega} . \nabla \times \boldsymbol{\omega},$$

where $\omega = \nabla x \mathbf{u}$ is the vorticity. Experiments have so far proved unable to determine whether, as theory suggests, the average value of this quantity over the fluid is zero when certain symmetry conditions are met by the convection chamber geometry, and finite otherwise.