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A NUMERICAL INVESTIGATION OF THE FLOW IN A FULLY BLOCKED DIFFERENTIALLY HEATED ROTATING FLUID ANNULUS

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ABSTRACT

A computational model has been used to determine the terms in the dynamical equations which are responsible for the formation of a certain horizontal circulation seen in a simply connected, differentially heated rotating fluid annulus. Experiments with a differentially heated rotating fluid annulus that is fully blocked by a thin, rigid, vertical radial barrier at rotation rates of up to 5 rad.sec^{-1} and with an externally applied radial temperature difference of 4 or 10 °C show two principal circulations. This paper is concerned with the mechanism for one of those circulations, which occurs in a horizontal plane. Computer modelling shows that this circulation is caused by small radial temperature gradients in the fluid, rather than centrifugal effects.

1. Introduction

The study of rotating differentially heated fluids is of importance to the understanding of geophysical systems such as the atmosphere and oceans[1]. The differentially heated, rotating fluid annulus shares rotational and thermal forcing with these systems. Also it has the advantage of simple, well defined boundary conditions. Experiments have been carried out[2] using the annulus shown in Figure 1(a). Fluid was trapped between two co–axial cylinders and a rigid thermally insulating lid and base. The annulus was placed upon a uniformly rotating turntable (with angular velocity Ω) so that its axis of symmetry was coincident with the axis of rotation. Thermal forcing was provided by maintaining the side walls of the annulus at two different, uniform temperatures, T_a and T_b.

The fluid heat transport through such a system decreases with increasing Ω[1]. However when the annulus was fully blocked by a thin radial barrier the heat advection became almost independent of rotation rate[2,3]. The processes responsible for this have been investigated[2]; velocity measurements showed the two main circulations which arose when the annulus was fully blocked (see Figure 1(b)). The present paper summarizes the results of the computational investigation of the horizontal circulation.

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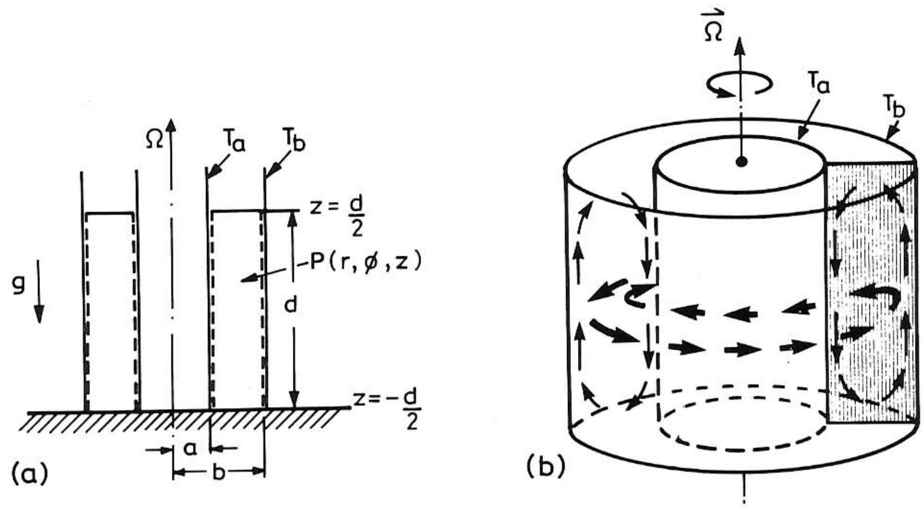

Fig. 1. (a) Diagram of fluid annulus. (r, ϕ, z) are cylindrical polar coordinates of a general point P, fixed in a frame moving with the annulus which rotates uniformly at Ω rad.sec^{-1}. The upper surface is in contact with a rigid lid[8]. $\Delta T \equiv T_b - T_a = 4$ or $10°$C. $a = 2.5$cm, $b = 8.0$cm, $d = 14.0$cm. (b) Diagram illustrating the two main circulations seen when the annulus is fully blocked by a thin radial barrier[2]. The side–walls are at constant temperatures T_a and T_b, with $T_b > T_a$.

2. The Computer Model

The numerical model of the annulus used here is one which was used within the U.K. Meteorological Office[4,5,6]. It is a grid-point finite difference formulation based on the Navier–Stokes equations for incompressible baroclinic flow in a Boussinesq fluid. The standard resolution of the model is 16 (vertical) by 16 (radial) by 64 (azimuthal) points; the grid is stretched in the (r, z) plane to resolve boundary layers. The dynamical equations used by the model[5] are:

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} + 2\vec{\Omega} \times \vec{u} - (\vec{g} - \Omega^2 r)\alpha(T - T_0) + \frac{1}{\rho_0}\nabla p = \vec{F}$$

$$\nabla.\vec{u} = 0$$

$$\frac{\partial T}{\partial t} + \vec{u}.\nabla T = \nabla.(\kappa \nabla T)$$

where $\vec{u}$ is the fluid velocity, t is time, $\vec{\Omega} = (0, 0, \Omega)$, $\vec{g} = (0, 0, g)$ is the acceleration due to gravity, $(T - T_0)$ represents the deviation of temperature, T from some reference temperature, T_0, α is the coefficient of thermal expansion for the fluid, κ is the thermal diffusivity, and $\vec{F}$ represents viscous forces. The equation of state is $\rho = \rho_0[1 - \alpha(T - T_0)]$ where the fluid density, $\rho = \rho_0$ when $T = T_0$. p is the deviation of the pressure from the reference function $p_0(z,r) = \rho_0 g(d-z) + \frac{1}{2}\rho_0\Omega^2 r^2$ where d is the depth of the annulus.

The model has previously been applied to unblocked flows[6]. Recently it has been modified to include a fully blocking thermally insulating barrier[7].

3. The Effect of the Centrifugal Force

A horizontal circulation in the same sense as that in the fully blocked annulus[2] has previously been observed in a rotating rectangular tank in which the end–walls were differentially heated[9]. In this case the horizontal circulation was shown to be due to the curvature of isopotential surfaces caused by centrifugal forces.

Observations of fluid velocities and temperatures in the annulus[2] showed that the horizontal circulation was in the sense consistent with this interpretation. To test the idea that the centrifugal force was responsible for the horizontal circulation two identical simulations of the annulus were carried out, one using the full dynamical equations, and the other with the centrifugal force term (but not the Coriolis term) removed. The results are shown in Figure 2(a) and (b). There is no discernible difference between the model results, thus the centrifugal force plays no significant role in the formation of the horizontal circulation.

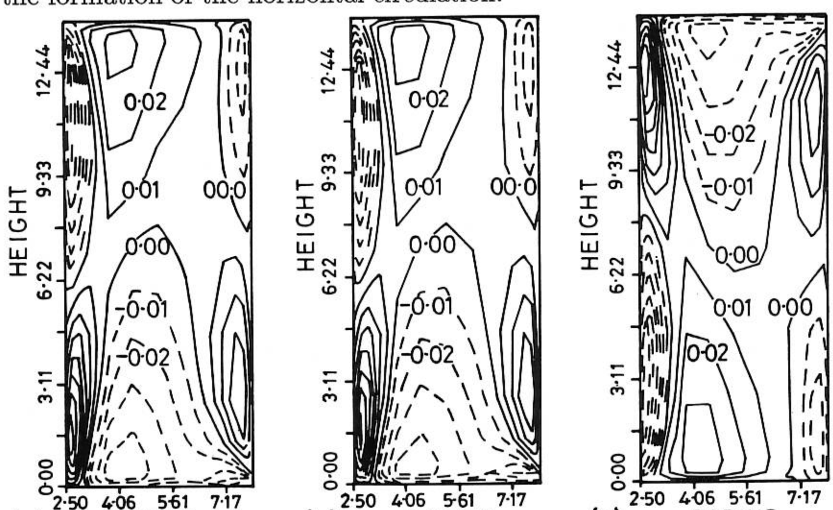

Fig. 2. Computer model results with $\Omega = 1.2$ rad.sec^{-1}. Cross–sections in the (r, z) plane showing mean azimuthal velocity, v (cm.sec^{-1}). Solid contours denote $v \geq 0$, dashed contours $v < 0$. (a) Run with the full dynamical equations and $\Delta T = +4°$C. (b) Run with the centrifugal force term omitted and $\Delta T = +4°$C. (c) Run with the full dynamical equations and $\Delta T = -4°$C.

The difference between this result and that of Condie and Griffiths[9] probably arises because the smaller radial dimension of the annulus used here (2.5cm $\leq r \leq$ 8.0cm) compared with the rectangular tank ($r \leq$ 100cm) means that the centrifugal force term ($\Omega^2 r$) is less important in the annulus.

4. The Effect of Radial Temperature Gradients

The values of Ω and ΔT applied to the annulus were such that geostrophic balance holds. In this case

$$\frac{\partial v}{\partial z} \approx \frac{g\alpha}{2\Omega} \cdot \frac{\partial T}{\partial r}$$

where v is the azimuthal fluid velocity, z is the height, and r is radius.

Reversing the sense of ΔT should reverse the radial temperature gradient in the fluid, and result in a reversal of the sense of v at a given height in the annulus. However against this, experimental work[3] has indicated that in the fully blocked annulus, in the body of the fluid, $\partial T/\partial r$ is much smaller than either $\partial T/\partial z$ or $\partial T/\partial \phi$, suggesting that it is not important in determining fluid motions. To resolve this question, two computer simulations of the flow in the annulus were executed under identical conditions except for reversing ΔT. The results are shown in Figure 2(a) and (c). The sense of the horizontal circulation has reversed, revealing that the radial temperature gradients in the fluid are responsible for the formation of the circulation through radial geostrophic balance. Other differences between the two sets of results arise because of the cylindrical geometry of the system.

5. Conclusions

Computer simulations of the flow in a fully blocked differentially heated rotating fluid annulus have shown that the horizontal circulation seen in the system is not due to centrifugal effects (as might have been expected based on the conclusions of earlier work[9]), but arises as a result of radial temperature gradients in the fluid, coupled with radial geostrophic balance.

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