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SAINT LOUIS UNIVERSITY
PARKS COLLEGE

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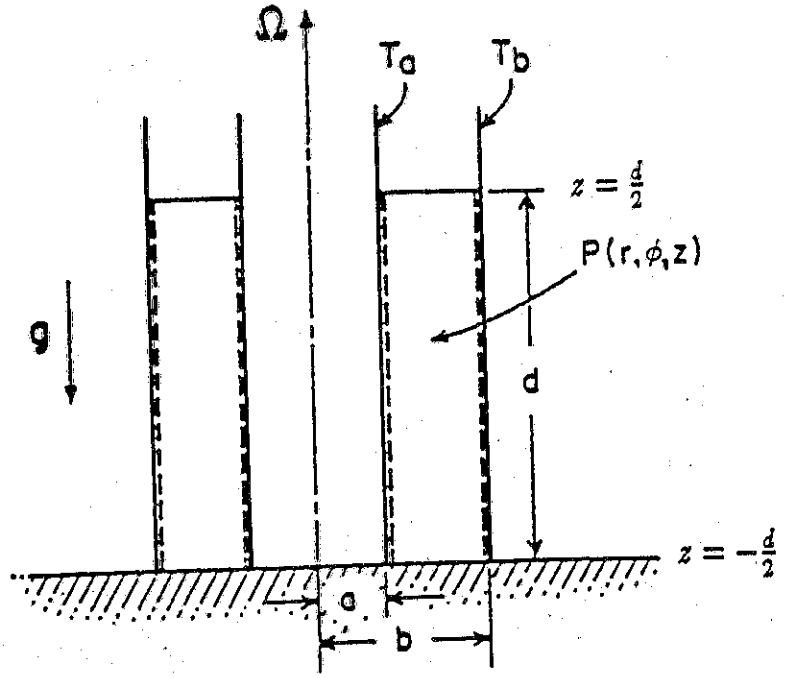
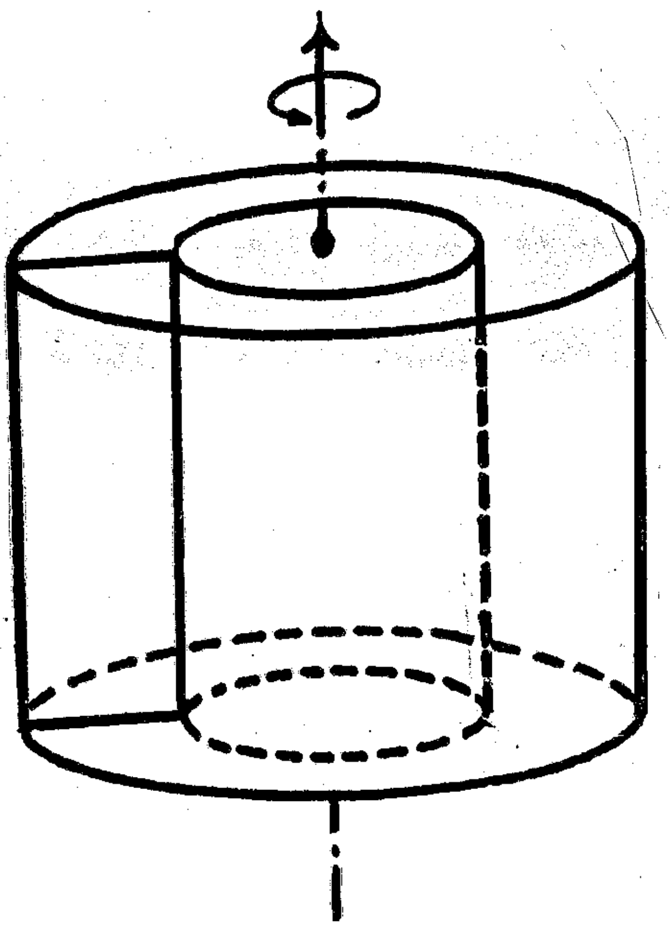
PS 12A Continuum/Fluid Mechanics

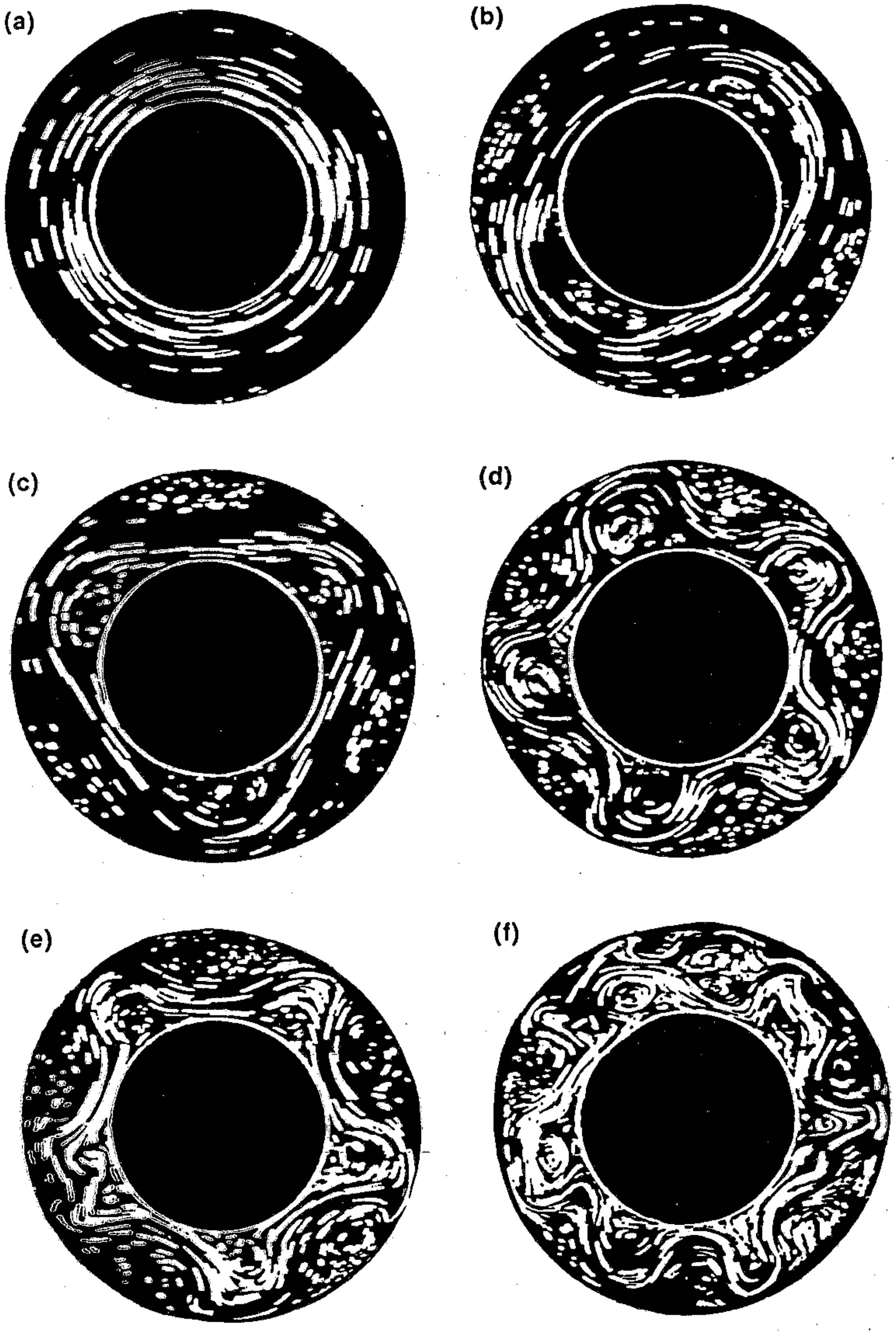
A Numerical Investigation of the flow in a fully blocked differentially heated rotating fluid annulus

Overview of talk

- Description of the rotating annulus, including the barrier
- Background: flow in an unblocked annulus, and the effect of the barrier on the fluid heat transport
- Experimental results of flow with the barrier
- Use of a computer model to investigate the flow seen with the barrier

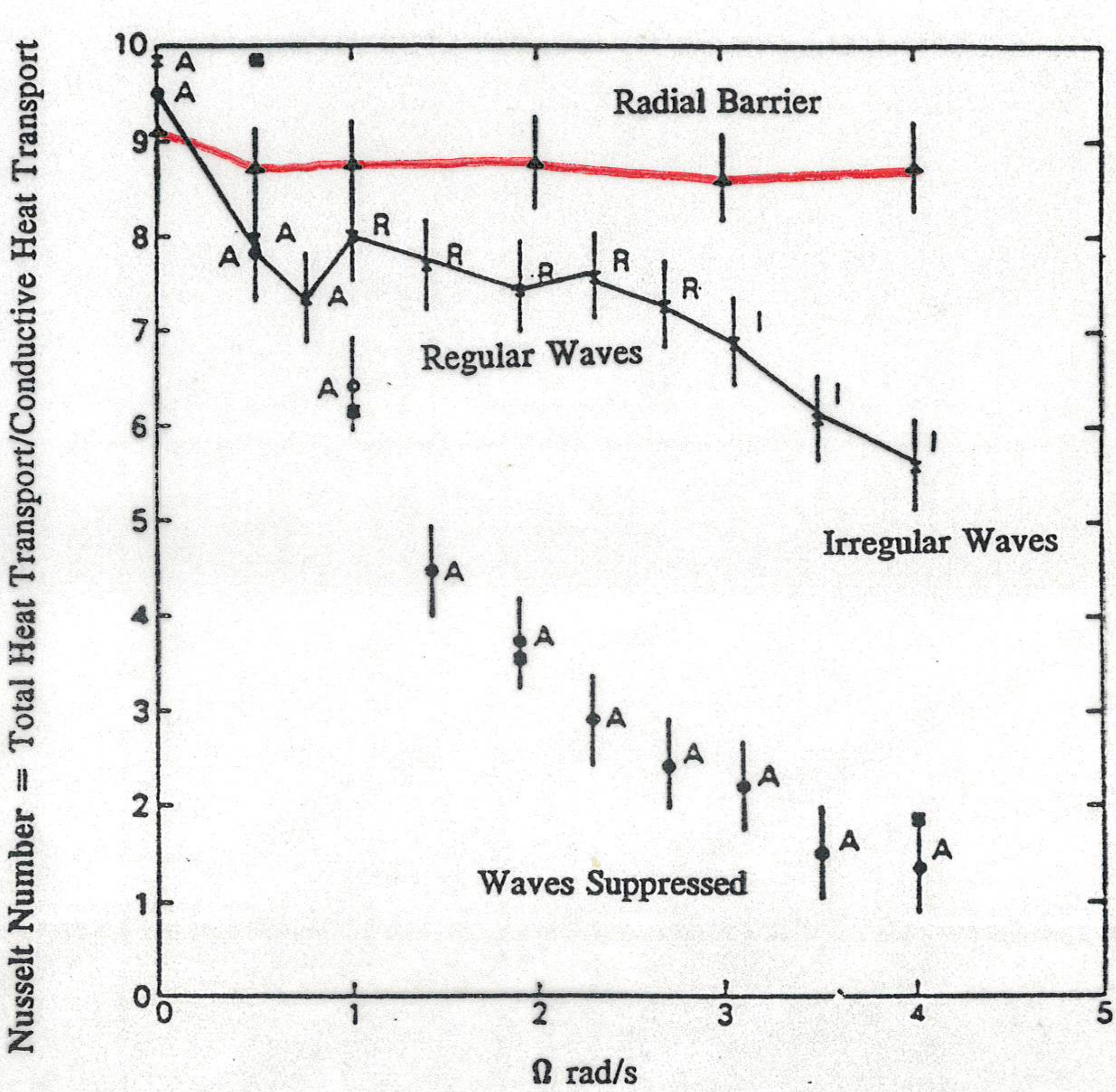

Ω
Ta
Tb
z = d/2
P(r, φ, z)
g
d
z = −d/2
a
b





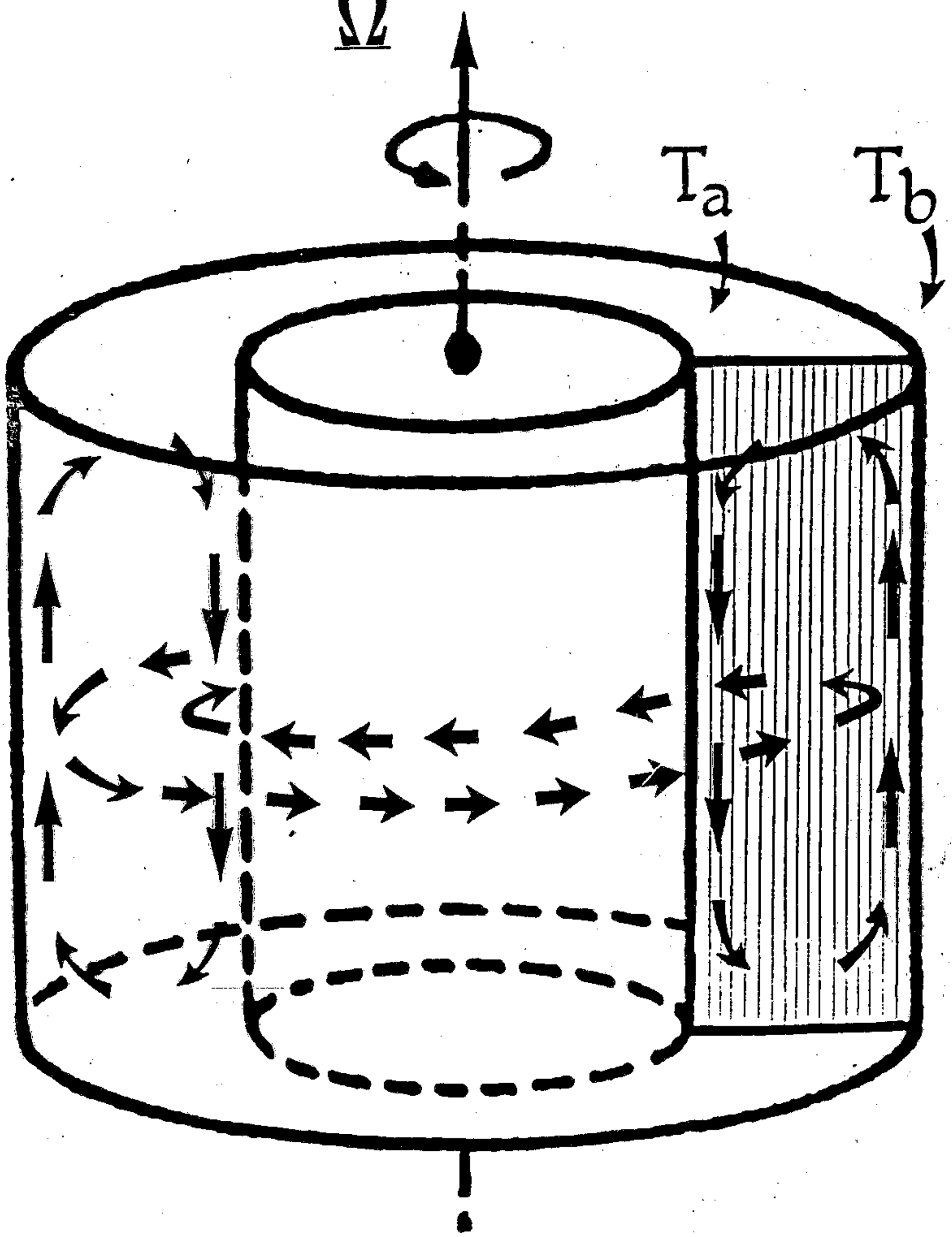
Read (1988): Surface flow patterns. (b-a)=4.6cm, d=13.5cm, $\Delta T=9°C$. Values of Ω: (a) 0.41 rad/s; (b) 1.07 rad/s; (c) 1.21 rad/s; (d) 3.22 rad/s; (e) 3.91 rad/s; and (f) 6.4 rad/s.

RADIAL BARRIER.



Radial Barrier:

$$Nu(\Omega \neq 0) \simeq Nu(\Omega = 0)$$



T_b-T_a=4 or 10°C

Dynamical Equations

In cylindrical polar coordinates with position vector $\underline{r}$, fluid velocity $\underline{u}$ and rotation rate $\underline{\Omega}$,

$$\underline{r}=(r,\phi,z) \qquad \underline{u}=(u,v,w) \qquad \underline{\Omega}=(0,0,\Omega)$$

Navier-Stokes Equation in a rotating frame, using Boussinesq approximation

$$\frac{\partial \underline{u}}{\partial t}+(\underline{u}.\nabla)\,\underline{u}+2\underline{\Omega}\times\underline{u}=-\frac{1}{\rho_0}\nabla p+\nabla\Phi-\alpha\,(T-T_0)\,\nabla\Phi+\nu\nabla^2\underline{u}$$

where

$$\nabla\Phi=\underline{g}-\underline{\Omega}\times(\underline{\Omega}\times\underline{r})+\underline{F} \qquad [where \quad \underline{g}=(0,0,-g)]$$

and

$$\nabla.\underline{u}=0 \quad , \quad \frac{\partial T}{\partial t}+\underline{u}.\nabla T=\nabla.(\kappa\nabla T)$$

Equation of state

$$\rho=\rho_0[1-\alpha(T-T_0)]$$

Neglecting Centrifugal effects, and scaling leads to geostrophic balance and the two components of the 'thermal wind' equation:

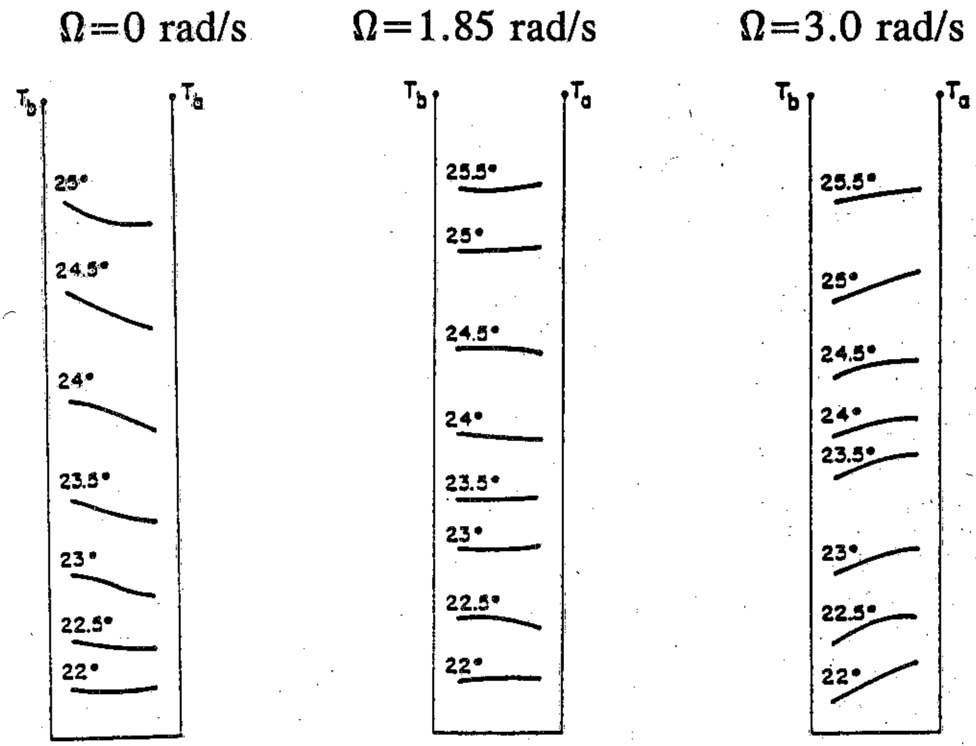
$$\frac{\partial v}{\partial z}\approx\frac{g\alpha}{2\Omega}.\frac{\partial T}{\partial r}$$

$$\frac{\partial u}{\partial z}\approx-\frac{g\alpha}{2\Omega r}.\frac{\partial T}{\partial \phi}$$

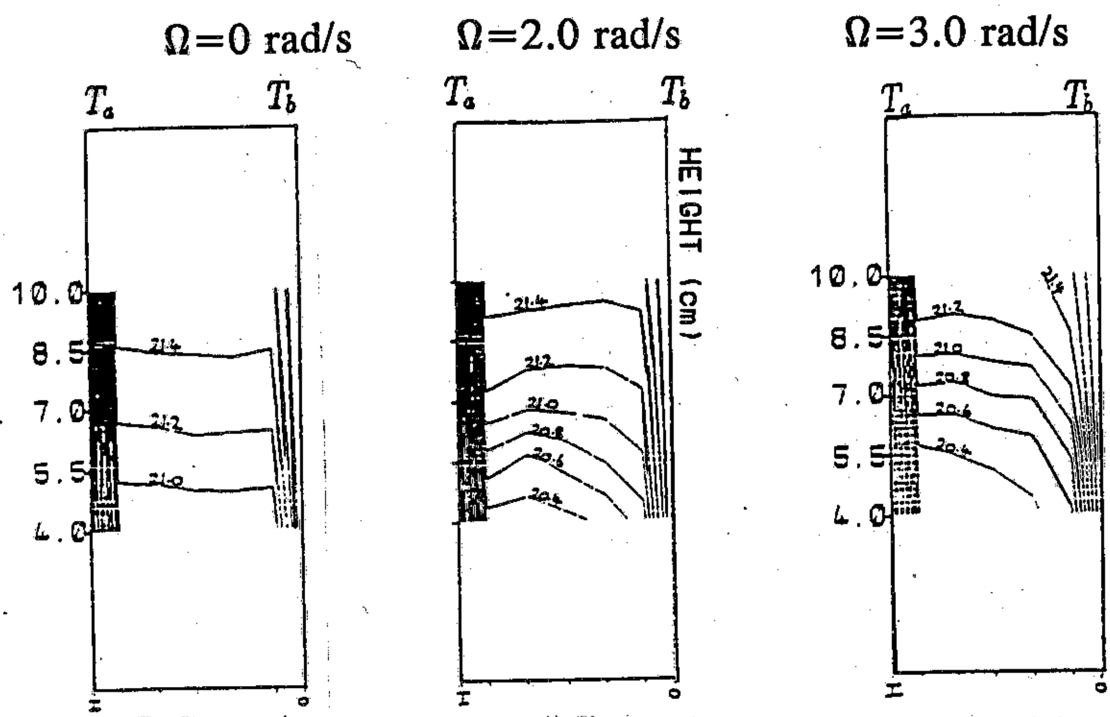
Including Centrifugal effects,

$$\frac{\partial v}{\partial z}\approx\frac{g\alpha}{2\Omega}.\frac{\partial T}{\partial r}+\frac{\Omega r\alpha}{2}.\frac{\partial T}{\partial z}$$

$$\frac{\partial u}{\partial z}\approx-\frac{g\alpha}{2\Omega r}.\frac{\partial T}{\partial \phi}$$

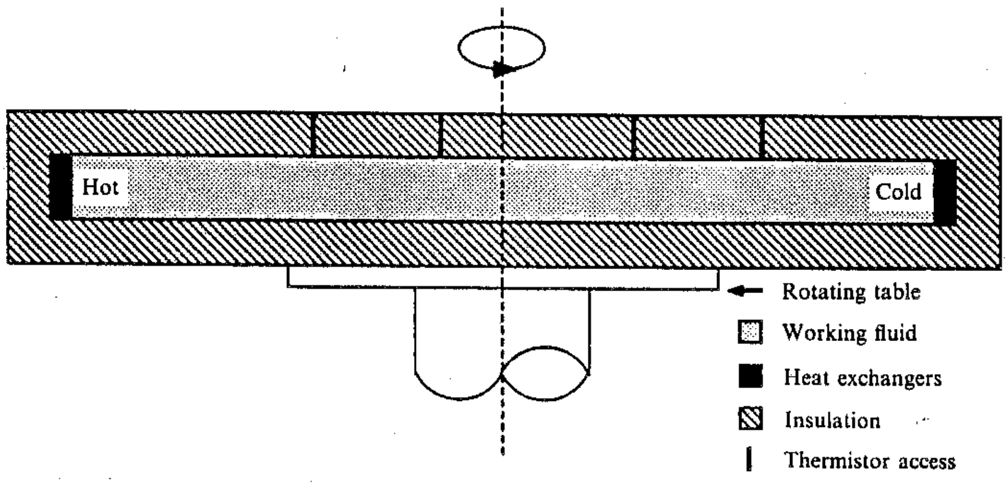


Bowden and Eden (1968): a=3cm, b=5cm, d=10cm, $\Delta T=6°C$

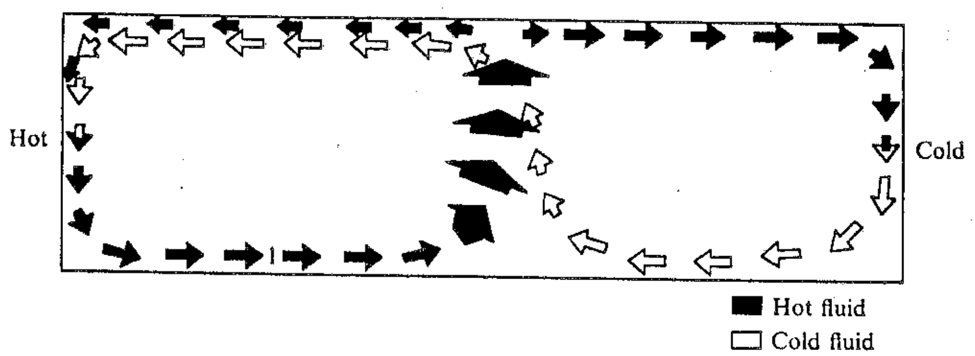


D.W.Johnson (private communication): a=2.5cm, b=8.0cm, d=14cm, $\Delta T=4°C$

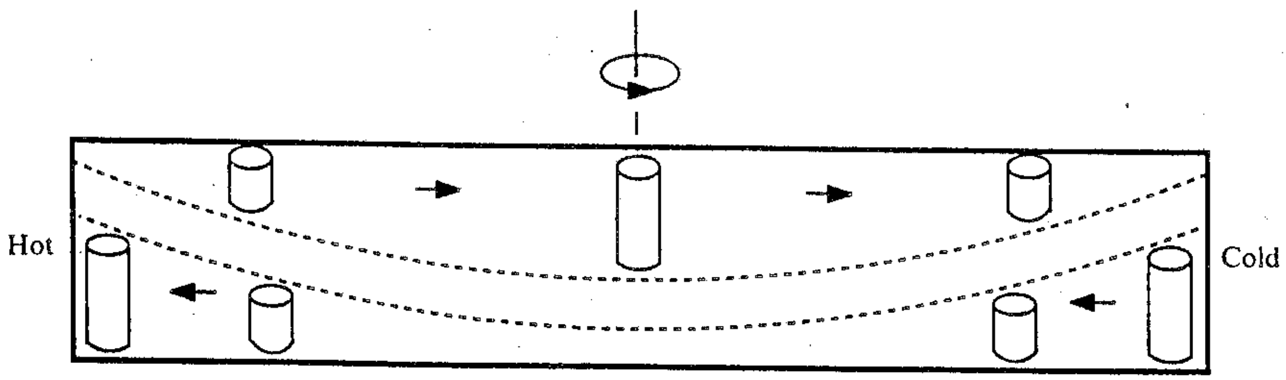
Condie & Griffiths (1989):



(a) Side view of tank.



(b) Plan view showing flow.



(c) Side view showing surfaces of constant potential.

Computer Model

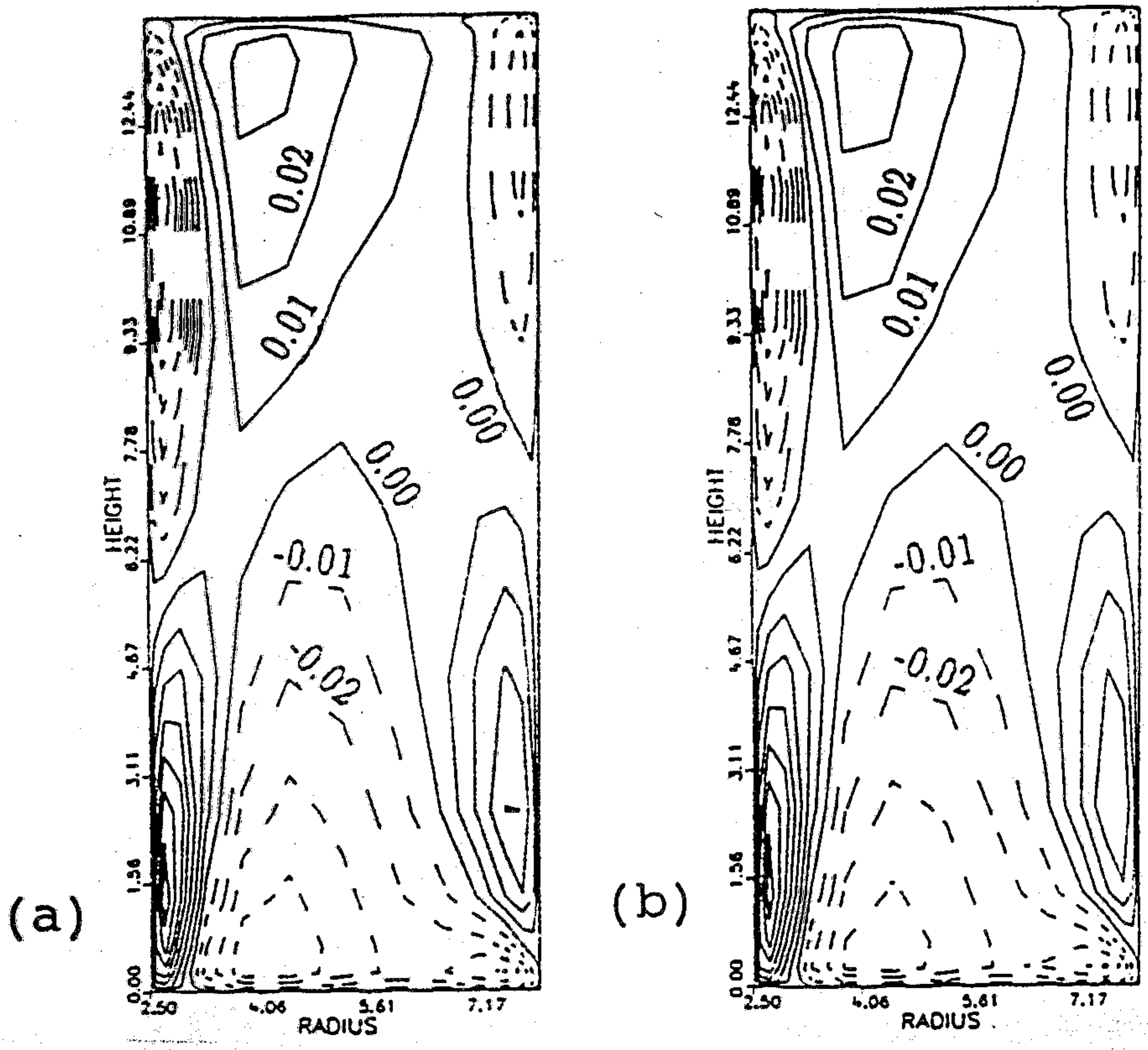
A finite difference formulation based on the (non-hydrostatic) Navier-Stokes equations for incompressible flow in a Boussinesq fluid.

Model Resolution: 16 vertical points
16 radial points
64 azimuthal points

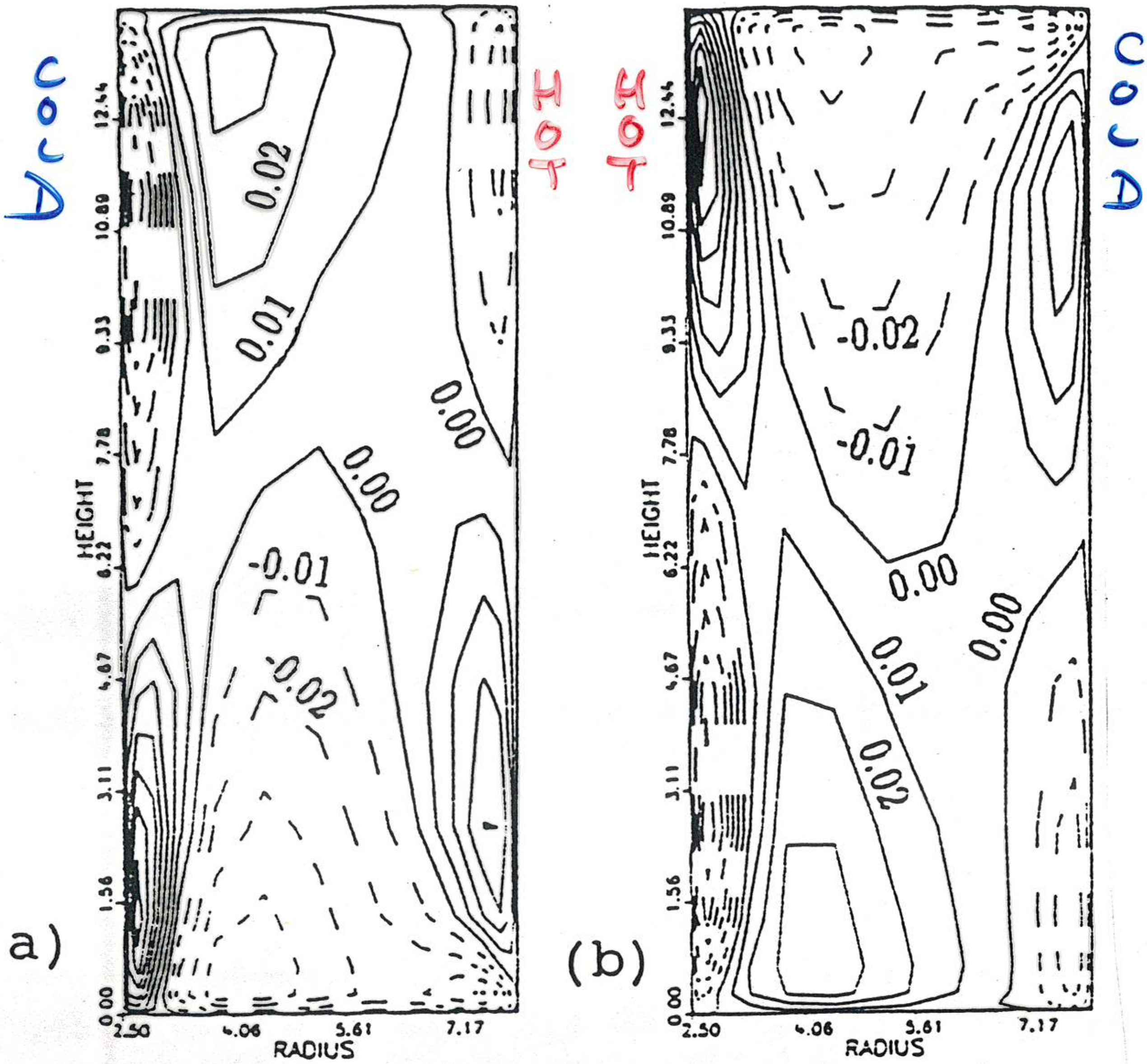
Grid stretched in (r,z) plane to resolve boundary layers.

Boundary Conditions: 'no slip' condition

Previously applied to annulus flows without a radial barrier, more recently modified to include barrier.



Computer model results with $\Omega = 1.2$ rad/sec, $\Delta T = 4°C$. Cross-sections in the (r,z) plane showing mean azimuthal velocity, v (cm/sec). Solid contours denote $v \geq 0$, dashed contours $v < 0$. (a) Run with the full dynamical equations, (b) run with the centrifugal force term omitted.



(a) (b)

Computer model results with $\Omega = 1.2$ rad/sec. Cross-sections in the (r,z) plane showing mean azimuthal velocity, v (cm/sec). Solid contours denote $v \geq 0$, dashed contours $v < 0$. (a) Run with $\Delta T = +4°C$, (b) run with $\Delta T = -4°C$.

Conclusions

- Horizontal circulation not due to Centrifugal effects

- But is caused by radial temperature gradients in the fluid coupled with radial geostrophic balance:

$$\frac{\partial v}{\partial z} \approx \frac{g\alpha}{2\Omega} \cdot \frac{\partial T}{\partial r}$$