2nd IMACS conference talk. Title: A numerical investigation of the flow in a fully blocked differentially heated rotating fluid annulus. Given 6/10/93.

1 Title slide

2 I am currently based at Nuclear Electric's Berkeley Technology Centre in Gloucestershire.

This talk is based on work from my D Phil thesis, when working at Robert Hooke Institute.

The Robert Hooke Institute was involved in research in geophysical fluid dynamics.

3 Talk concerns computational investigations that arose as a result of experiments carried out with a differentially heated rotating fluid annulus.

I shall start by giving a description of the annulus, and of the barrier that was used,

also some background, which includes a brief summary of the flows seen in an annulus without a barrier.

This helps explain interest in the effect of the barrier on the flows, including the effect it has on the fluid heat transport.

A summary of the experimental results is used to show the effect the barrier has on the flow.

The computer model is used to investigate an aspect of the flow observed in the experiments.

4 The annulus consists of two concentric cylinders which are used to form an annular convection chamber.

Fluid is trapped between an insulating lid and base. The outer wall is heated to a constant temperature T_b and the inner wall cooled to a constant temperature T_a.

The whole thing is then placed on a turntable, so that the central axis of symmetry coincides with the axis of rotation, and is rotated with uniform angular velocity.

Dimensions	Inner radius	2.5 cm
	Outer radius	8.0 cm
	Depth	14.0 cm

Experimental parameters

T_b-T_a=ΔT=4 or 10 °C

Ω=0 to 5 rad/sec

This is not Taylor-Couette flow; the two cylinders, lid and base all rotate at the same rate; they are physically connected.

The investigation discussed here includes a thin, thermally insulating radial barrier, illustrated in the lower part of the slide.

5 To help understand interest in the effect of the barrier on the flow, I shall outline the observed flows when no barrier is present.

In this case the flow generally proceeds around the annulus. At low rotation rates the flow is axially symmetric, with regular waves forming at higher rotation rates.

The wave number increases as the rotation rate increases, until the waves become irregular at the largest rotation rates.

Since the flow is largely azimuthal it is natural to ask what happens if we block it?

6 This slide shows experimental results of the fluid heat transport (Nusselt Number) against rotation rate (Ω).

$$N = \frac{\text{Total heat transport}}{\text{Heat conduction}}$$

The solid line shows results for an unblocked annulus. Heat transport decreases rapidly ($\propto \Omega^{-3/2}$) at first, and then picks up (to $\sim$75-80% of the non-rotating value) when the regular waves start, before gradually trailing off at higher Ω.

PTO

It is clear that the regular waves carry heat, because if you suppress them, then you get the lower line, where the heat transport just continues to diminish.

Putting in a radial barrier has the effect of keeping the heat transport very close to its non-rotating value over all Ω.

This prompts the question: how does the fluid do this? So we want to understand what causes the flows in the annulus with a barrier.

7 This slide summarizes the flows seen in the experiments with a barrier. It is a simplification.

Basically there are two main circulations: a radial overturning cell, and a horizontal circulation.

This talk is concerned with finding the mechanism for the horizontal circulation. Experimental investigations have shown the processes responsible for the radial overturning cell.

So what causes the horizontal circulation?

8 Start by looking at the equations of motion. Use cylindrical polar coordinates, with position vector $\underline{r}$ and components (r,ϕ,z); velocity $\underline{u}$, components (u,v,w) and rotation vector $\underline{\Omega}$, which is $(0,0,\Omega)$.

The motion of a fluid particle is described by the Navier-Stokes equation, in a rotating frame.

Buoyancy effects are included by using the Boussinesq approximation, where horizontal density variations are only considered when coupled with gravity.

Terms are: Acceleration relative to a fixed point

Inertial acceleration, which together with the first term, makes up the acceleration of a fluid particle following the motion of the fluid

Coriolis acceleration

Pressure gradient acceleration

Potential of external forces, $\nabla\Phi$, which includes gravity, the centrifugal acceleration, and potentially any other conservative forces acting on the fluid

Buoyancy

PTO

Viscous term, based on a Newtonian Fluid, i.e. constant viscosity ν.

Mass continuity equation, assumes an incompressible fluid.

Also have the equation of heat transfer, can see terms for heat advection and conduction.

In the equation of state density is linearly dependent on temperature, using the coefficient of thermal expansion, α.

It is possible to scale these equations to show which of the terms are significant.

Put in order of magnitude values for typical length scales, fluid velocities etc.

When you do this, for the interior of the fluid, you are left with the Coriolis term, pressure gradient term, and terms with gravity in them.

Can split the terms into their components and cross-differentiate to eliminate the pressure.

This gives the components of the so-called 'Thermal Wind' equation, in which horizontal temperature gradients are related to a vertical shear in velocity. This equation also applies to atmospheric motions.

In the derivation of the Thermal Wind equation, it is usual to exclude the centrifugal acceleration, but it can be included.

Thus the azimuthal component of the velocity, v making up the horizontal circulation seen in the experiments (with a barrier) is likely to be linked to either radial temperature gradients ($\partial T/\partial r$) or else to centrifugal effects.

9 Considering that a radial temperature difference was applied to the convection chamber, you might expect the radial temperature gradient in the fluid to be quite big.

But experimental measurements show that it is not.

The values of $\partial T/\partial r$ with a barrier are much less than those seen in an unblocked annulus.

The slide shows isotherms for flow in an annulus with a barrier, from two sets of measurements.

In the more recent results (shown in the lower part of the slide), there are large values of $\partial T/\partial r$ in the boundary layers, but the Thermal Wind equation doesn't apply there anyway.

So radial temperature gradients don't seem a very promising source for the horizontal circulation at this stage. What about the centrifugal force?

10 There was some evidence to support the idea that the centrifugal force might be important.

Condie & Griffiths had done some experiments with a rotating rectangular tank (2m long), differentially heated along its narrower sides.

They observed a circulation in the horizontal, which formed a figure of eight.

They suspected that the centre of this circulation occurred at the point where the parabolic surface of constant potential in the fluid was parallel to the base of the tank. (The constant potential surface being parabolic because of the centrifugal forces).

So they sloped the base of the tank, and found that the centre of the circulation moved to the new location where the surface of constant potential was parallel to the sloping base.

This showed that their horizontal circulation was caused by centrifugal forces.

11 With a computer model you can do something that's impossible in an experiment. Namely have rotation without the centrifugal force. i.e. modify the equations so that the centrifugal term is removed, but the Coriolis acceleration is kept.

So one can do two computer simulations, one with the centrifugal force, and one without, to see what difference it makes.

First, a few details about the model are on the slide.

It had been used for some time at the UK Met. Office for modelling unblocked flows.

More recently it was modified to include the barrier, but not much work had been done with it.

It seemed a good opportunity to try out my idea.

12 Slide of sets of model results. Shows a cross-section in the (r,z) plane.

Plotted are contours of the mean azimuthal component of velocity, v.

Solid contours : $v \geq 0$
Dashed contours: $v < 0$

(a) Full equations
(b) Centrifugal force term omitted

Hence centrifugal force makes no appreciable difference.

So is the horizontal circulation caused by radial temperature gradients then?

Test this by running two simulations, identical except that the <u>sense</u> of the externally applied temperature difference has been reversed.

13 Same sort of plot as before.

(a) $T_b - T_a = +4°C$
(b) $T_b - T_a = -4°C$

One can see that the sense of the mean azimuthal component of the velocity, v has very neatly reversed.

The two sets of results are not a mirror image of one another because of the cylindrical geometry of the annulus.

So the horizontal circulation must be caused by radial temperature gradients in the fluid.

14 Summary of results.