Chapter 8

Conclusions.

8.1 Full barrier experiments.

This section summarizes the conclusions of chapters 3 to 6 above.

As a consequence of the work of *Bowden and Eden (1968)* and others, described in chapter 1, this thesis has attempted to answer the three questions put forward in §1.1.1 (p10).

In chapter 3, three significant components of the flow in a fully blocked annulus were identified. These were the η and ζ–circulations (shown in *Figure 3.12* and described in §3.2.1), and the eddies seen at higher Ω. The η–circulation corresponds to the mechanism mentioned by Hide (§1.3.2), which he suggested was primarily responsible for keeping the heat transport largely independent of Ω. Each of these processes is discussed in §§8.1.1 to 8.1.3 below.

8.1.1 The η–circulation.

The mechanism for the η–circulation is described in detail in §3.2.3. In brief, it describes the dependency of $u(z)$ on $\partial T/\partial\phi$ based on one of the components of thermal wind equation (1.11). Thus so long as $\partial T/\partial\phi \propto \Omega$, $u(z)$ may be independent of Ω, and consequently the heat advection may also be independent of Ω. *Figure 3.10* shows ΔT_B (which may be regarded as being related to the average $\partial T/\partial\phi$ in the fluid) plotted against Ω. Since ΔT_B does not increase in

proportion to Ω indefinitely there comes a point where $u(z)$ begins to decrease with increasing Ω, and a corresponding decrease in the heat advection by the η–circulation is also to be expected, proportional to Ω^{-1} for constant ΔT_B.

Figure 3.17 shows the heat advection calculated for the η–circulation based on a simple linear model, plotted against Ω, and the measured heat advection of the fluid. It can be seen that the η–circulation accounts for the greater part of the fluid heat transport over a considerable range of Ω. Also, at high Ω, H_η decreases with Ω as expected from the considerations mentioned above.

It seems likely that the discrepancies seen between H_η based on the simple model and the measurements arise because of the simplifications in the model of the velocity and temperature fields; in particular the estimate used for ΔT_z and the assumption that $\partial T/\partial\phi$ is independent of z. It is also to be expected that at very small Ω the model should break down because azimuthal geostrophic balance no longer applies. The dependence of $\partial T/\partial\phi$ on z could be very significant, because if $\partial T/\partial\phi$ were to increase towards the top and bottom of the annulus, this would imply the existence of 'jets' near the top and bottom, where u would be greater than the values calculated for the linear model mentioned above. This would tend to increase the heat advection.

The values of $\partial T/\partial\phi$ seen in the experiments were associated with a temperature drop across the barrier, ΔT_B. In chapter 4 an attempt was made to remove, or reduce ΔT_B by using a barrier made of a thin sheet of copper. If ΔT_B had been significantly affected, a concomitant modification to $\partial T/\partial\phi$ would also have been expected, with consequences for the η–circulation. However the fluid was able to support ΔT_B, despite the barrier being an excellent conductor. If ΔT_B had been applied directly across the barrier, the heat conduction would have been enormous, this suggests that there may be some boundary layer structure by the side of the conducting barrier. Order of magnitude calculations (§4.2) suggested that if there was a boundary layer 5mm thick to each side of the barrier (so that the boundary layer would be thin enough to remain undetected by the nearest thermocouple), the w associated with a buoyancy–viscous balance would be sufficient to advect away the heat conducted through the boundary layer.

The model of the η–circulation described in chapter 3 gave less good agreement for the measurements with the conducting barrier, than with the insulating barrier; however the discrepancy was such that it suggested that ΔT_z in the conducting barrier experiments was different from the insulating barrier experiments, with a value of $\Delta T_z \sim 0.8\Delta T$. For the insulating barrier experiments a value of $\Delta T_z \sim \Delta T$ had seemed appropriate.

In the experiments reported in chapter 6, sloping bases were used to attempt to suppress the formation of the eddies seen at large Ω. In the case where one of the sloping bases did suppress the formation of the eddies, *Figure 6.13 (a)* the agreement between the heat advection calculated from the simple model involving the η–circulation and the measured heat advection was extremely close. This suggests that the linear model of the η–circulation was essentially correct.

Thus it appears that the η–circulation makes a significant contribution to the heat advection by the fluid at low to medium Ω, in the range where $\Delta T_B \propto \Omega$. Beyond this range its heat transport contribution decreases, as other processes become more important.

Thus the answer to the first question of §1.1.1 is that radial geostrophic flow is probably responsible for making the fluid heat transport largely independent of Ω, over low to medium Ω, so long as $\Delta T_B \propto \Omega$.

8.1.2 The ζ-circulation.

The ζ–circulation is described in §3.2.1 and illustrated in *Figure 3.12*. In chapter 3 no mechanism is suggested for the ζ–circulation, but its heat advection is estimated from the velocity measurements using the incompressibility equation (1.2), using a method which was more likely to lead to an over–estimate of H_ζ than an under–estimate. Even so, *Figure 3.17* shows that the ζ–circulation was unable to account for the discrepancy between the η–circulation and the heat transport

measurements at high Ω.

In chapter 5 two possible mechanisms for the ζ–circulation were advanced. The first involved a correction to radial geostrophic balance (equation (5.3)) to allow for the centrifugal force. Physically this can be interpreted as denser fluid on the cold side of the barrier being flung outwards by the centrifugal force, accompanied by a return flow of less dense fluid on the warmer side of the barrier. Clearly if the annulus is rotated at increasingly faster speeds there must come a stage where such an effect will become significant. This mechanism for the ζ–circulation was tested by running a numerical simulation of the fully blocked annulus twice. Both the simulations used were identical except that one had the centrifugal force term removed from the model. As there was no appreciable difference between the simulated flows, it appears that the centrifugal force does not play any significant part in the dynamics of the ζ–circulation. This test was not as decisive as would have been desirable, because the numerical model did not simulate the ζ–circulation particularly well in the first place. Though it did at least produce a weak ζ–circulation. However the model still served to show that the centrifugal force term was insignificant at that rotation rate.

The second mechanism proposed for the ζ–circulation was that of radial geostrophic balance, so that v arises through equation (1.10), a component of the thermal wind equation. Temperature measurements by *Bowden and Eden (1968)* had indicated that when the annulus was fully blocked $\partial T/\partial r$ was very small, however more recent measurements by D.W.Johnson show that there is quite a complex radial temperature structure in the body of the fluid, which though small compared with $\partial T/\partial z$ and possibly $\partial T/\partial \phi$, might still be large enough to account for the values of v seen in the experiments. If this is the case, then should the externally applied temperature difference, ΔT be reversed, $\partial T/\partial r$ would be expected to reverse also, and consequently v should change sign. *Figure 5.12* shows the result of the numerical simulation in which ΔT was reversed. Comparison with *Figure 5.3* shows that v has reversed with almost perfect symmetry about a line at mid-height. One would not expect the two figures to look like a 'negative' of one another other because of the cylindrical geometry of the system. Thus it appears that the ζ–circulation arises from radial geostrophic balance, coupled with the effect that the barrier has on $\partial T/\partial r$ in the body of the fluid.

8.1.3 The eddies.

In §3.2.6 an order of magnitude argument was used to show that if an eddy of the type seen in the velocity measurements was suitably correlated with the departures from linearity seen in the temperature measurements at the same Ω, then the eddy was capable of carrying significant quantities of heat. The estimation suggested that the eddies would certainly be capable of advecting sufficent heat to account for the discrepancy between the heat advection by the the η and ζ–circulations and the measurements. However as the temperature and velocity measurements were not made simultaneously, it was only possible to estimate the maximum amount of heat the eddy might be able to carry. A similar argument was used in §4.2.2 to show that the same result was equally valid for the conducting barrier.

In chapter 6 sloping bases were used to try to suppress the eddies, a technique which had proved successful with baroclinic wave flows in the past (see the preamble to chapter 6 for further details). Both the sloping base types used suppressed the formation of eddies to some extent, but the base which made $d = d(r,\phi)$ proved most successful, suppressing the eddies over the complete range of Ω used at $\Delta T \approx 4\ K$. The heat transport measurements with the $d(r,\phi)$ sloping base showed a decrease in Nu with increasing Ω, particularly in the case when the eddies were suppressed over the whole range of Ω (*Figure 6.8*). Two important conclusions can be drawn from this fact, by comparison with the heat transport results with the flat base in *Figure 3.11*, these are; the eddies transport heat and, they are suppressed by sloping bases. Both these characteristics are shared by baroclinic wave flows, suggesting quite strongly (but not proving conclusively)

that the eddies seen in the fully blocked annulus system are baroclinic in nature. This result seems more compelling when the manner in which the sloping bases affect the flow is considered (see the start of chapter 6 for a discussion), namely by influencing the angles of fluid parcel trajectories to the isotherms.

When the eddies were suppressed the measured heat transport showed good agreement with $H_\eta + H_\zeta$. This strongly suggests that heat advection by the eddies was responsible for the discrepancies in those heat transports seen at high Ω in the other experiments (chapter 3, *Figure 3.17*).

An interesting feature of the eddies is that though they appear at $\Omega \gtrsim 1.2\ rad.sec^{-1}$ (in the insulating barrier experiments at $\Delta T \approx 4\ K$) they do not appear to transport an appreciable amount of heat until $\Omega \gtrsim 3\ rad.sec^{-1}$, as can be seen from *Figure 3.17 (a)*. This is the value of Ω at which ΔT_B stops increasing with Ω, see *Figure 3.10*. Since the departures from linearity in $\partial T/\partial\phi$ appear to be associated with rotation rates at which ΔT_B is no longer proportional to Ω, this would suggest that significant heat transport by the eddies is associated with the 'kinks' in the thermocouple ring measurements, as suggested in §3.3.

8.1.4 Summary.

It appears that the η–circulation, ζ–circulation and the eddies are the three significant processes responsible for heat advection in the fully blocked annulus system.

At low to medium Ω (for Ω large enough for azimuthal geostrophic balance to hold) heat is advected mainly by the η–circulation, at a rate which is largely independent of Ω, so long as $\Delta T_B \propto \Omega$. The heat advection for the η–circulation is given by equations (3.10), (3.15) and (3.16); while $u(\bar{r}; z, t)$ is given by equation (3.14). The mechanism for the η–circulation is discussed in §3.2.3.

As Ω increases, ΔT_B increases and so does the strength of the ζ–circulation, so that heat advection by the ζ–circulation becomes increasingly important. However over the range of Ω covered by the experiments, $H_\zeta < H_\eta$ for all Ω. The heat advection by the ζ–circulation is given in equations (3.11), (3.21) and (3.23), and is discussed in §3.2.5. One mechanism for the ζ–circulation is described in §5.4.1 and shown to be inappropriate in §5.4.2. The second mechanism for the ζ–circulation is proposed in §5.4.3 and evidence is provided to support it.

The eddies appear at quite small values of Ω, but they do not seem to play a significant part in heat advection until the much higher values of Ω are reached, which are associated with the downturn in heat advection by the η–circulation. Heat advection by the eddies may be associated with the departures from linearity seen in $\partial T/\partial \phi$ at these rotation rates. Evidence involving the suppression of the eddies by sloping bases seems to suggest that they may be baroclinic in nature.

Thus in answer to questions 2 and 3 in §1.1.1; the ζ–circulation and eddies are responsible for keeping the heat transport nearly independent of Ω once the contribution from the η–circulation starts to diminish. There is evidence to suggest that the eddies are baroclinic in nature.

8.1.5 Other results.

The heat transport results of chapters 3 and 4 suggested that the increase seen in ΔT_B with the conducting barrier, was somehow at the expense of ΔT_z, so that the total heat transports of these two systems remained the same. This result was expressed in equation (4.1). Due to the approximations involved in obtaining that equation (that $H_{adv} \approx H_\eta$ in both cases) it is difficult to be certain how accurate it is. However it does suggest that simultaneous measurements of both ΔT_B and ΔT_z against Ω might prove worthwhile.

The result of *Bowden (1961)*, equation (1.19), was shown to correctly predict $\text{Nu}(\Omega = 0)$ for the fully blocked annulus measurements, for both the insulating and conducting barriers. Since for both of those sets of measurements, the heat transport was weakly dependent on Ω it is possible to obtain a similar expression for Nu for the range of Ω covered in the experiments. *Figure 8.1* shows plots of $\text{Nu.Ra}^{-1/4}$ for all the insulating and conducting barrier experiments of chapters 3 and 4. From the figure it can be seen that Nu for the insulating and conducting

barrier systems, for $0 \leq \Omega \lesssim 5.0\ rad.sec^{-1}$, and $\Delta T \approx 4, 10\ K$ is given by

$$\text{Nu}(\Omega) = (0.212 \pm 0.012)\text{Ra}^{\frac{1}{4}}.$$

Figure 8.1 also clearly shows the difference in the heat transport results between $\Delta T \approx 4\ K$ and $10\ K$. At $\Delta T \approx 10\ K$ there is an approximately linear looking dependence on Ω at high Ω. Since the upper limit of equation (1.19) is $\text{Nu.Ra}^{-1/4} = 0.213$, *Figure 8.1* clearly shows that at high Ω the heat transport by the fully blocked systems is greater than that expected for a non–rotating system.

If only the $\Delta T \approx 4\ K$ results are considered the error bar on Nu can be reduced to give $\text{Nu}(\Omega) = (0.205 \pm 0.005)\text{Ra}^{1/4}$.

8.2 Partial barrier experiments.

The velocity measurements with the $h/d = 2/3$ partial barrier suggested that the flow split into two regions; a lower region where the flow was very similar to that seen in the fully blocked system, and an upper region with strong azimuthal flow and a topographically forced wave. The heat transport measurements, and ΔT_B were analysed in the same manner as in chapter 3, and provided evidence to support the view that the flow in the lower, blocked region was similar to that seen in a fully blocked annulus.

$\text{Nu}(\Omega)$ was estimated for the upper, unblocked regions for the experiments with $h/d = 1/3$ and $2/3$ partial barriers. This was done by assuming that the heat advection in the lower blocked region was h/d times the heat advection in the full radial barrier experiments at the same Ω and ΔT. The resulting values of $\text{Nu}(\Omega)$ were (generally) qualitatively fairly similar to measurements of Nu in unblocked annulus systems. Detailed comparisons would require making suitable allowances for the heat transport of the topographically forced waves seen in the upper regions. Experiments were suggested that might allow these comparisons to be made. Measurements of v above the barrier were shown to be consistent with the interpretation that $\partial T/\partial r$ above the barrier had a value typical of the unblocked annulus.

The results seem to suggest that in a partial barrier annulus, with barrier height h and annulus depth d, the heat advection is approximately given by h/d times the heat advection of a similar fully blocked system, plus $1 - h/d$ times the heat advection of a similar unblocked system, at the same values of Ω and ΔT.

Comparison with the results of *Kester (1966)* and to a lesser extent *Leach (1975)* suggest that the transition for the onset of waves in the partial barrier system is affected more by the azimuthal width of the barrier than by its height.

8.3 Geophysical implications.

The work described in this thesis might be expected to have relevance to the dynamics of the oceans and the atmospheres of earth and possibly other planets. Once again it must be stressed that the annulus is in no sense an 'engineering model' of any geophysical system.

So far as oceanic flows are concerned there are many effects which are believed to be important, for which there are no analogues in these experiments. Examples include wind–stress, evaporation, salinity, the earth's curvature and the surface heating of the oceans. Complete or partial experimental analogues for many of these effects could be devised. Consequently it is not surprising that the currents of the North Atlantic (for example) show little sign of a circulation similar to the ζ–circulation (chapter 3). Indeed the Gulf Stream flows in the opposite direction. However further north, where the Coriolis parameter is larger, the Labrador current and the North Atlantic Drift could be interpreted as being part of a cyclonic circulation, the same sense as the ζ–circulation.

The partial barrier experiments might have some bearing on the Antarctic Circumpolar Current (ACC), where a strong zonal flow encounters topography. The results suggested that topography can effect the meridional heat transport by blocking the lower part of the fluid. The experiments also provide an opportunity

to test numerical models of the ACC. It seems likely that if an appropriately modified model cannot simulate the flow in a partially blocked annulus, it probably will not simulate the more complex circumstances of the ACC very well.

For the atmosphere, the partial barrier experiments suggest that topography could have a significant effect on the poleward transport of heat, by allowing a zonal pressure gradient to be set up, which would allow meridional geostrophic flow.

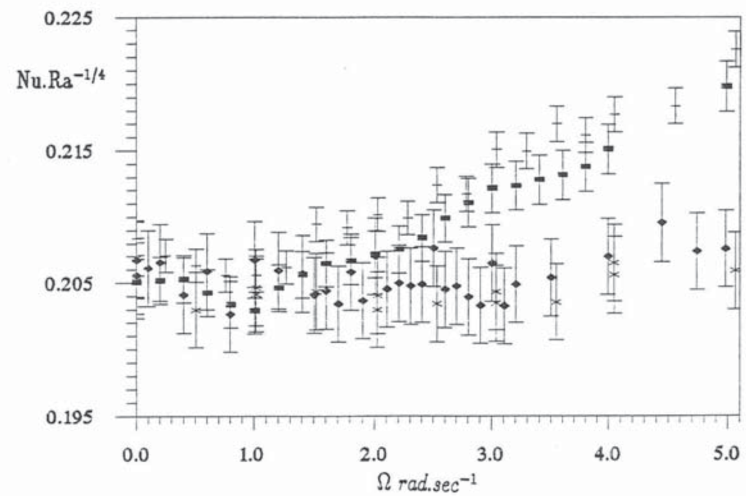

FIGURE 8.1: Plots of Nu.Ra$^{-1/4}$ against Ω for the fully blocking insulating and conducting barrier measurements. Key: solid diamonds, insulating barrier, $\Delta T \approx 4$ K; solid squares, insulating barrier, $\Delta T \approx 10$ K; diagonal crosses, conducting barrier, $\Delta T \approx 4$ K; orthogonal crosses, conducting barrier, $\Delta T \approx 10$ K. There is a clear divergence between the $\Delta T \approx 4$ K and 10 K results at high Ω.