

An experimental investigation of heat transfer by large scale motions in rotating fluids.

by

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An investigation has been made of the effect of barriers and sloping bases on the flow in a differentially heated, rotating fluid annulus.

The investigation of fully blocked systems has indicated the main processes that keep the fluid heat transport largely independent of rotation rate, up to 5 rad.sec^{-1} . The measurements show that for smaller rotation rates heat is advected by a radial geostrophic flow, with an azimuthal pressure gradient supported by the barrier. Associated with the pressure drop across the barrier, is a temperature drop, which remains even when the barrier is a good thermal conductor.

At higher rotation rates, two other heat advection processes become increasingly important; a horizontal circulation and eddies. Evidence that the horizontal circulation is not due to centrifugal effects is provided by a computer simulation of the flow. The simulation was also used to show that this circulation may be associated with radial temperature gradients in the fluid.

By sloping the base of the annulus it proved possible to suppress (to varying degrees) the formation of the eddies. The resulting heat transport measurements indicated that heat advection by eddies was significant at larger rotation rates. The heat transport measurements made with one of the sloping bases suggested that the eddies may be baroclinic in nature.

A second investigation explored the effect on the flow of barriers with a height less than the depth of the annulus. Measurements suggested that flow in the region blocked by the barriers was similar in character to the flow in the fully blocked annulus. Flow in the unblocked regions seemed to be more like that of an unblocked annulus perturbed by small topography. The heat transport measurements in the partially blocked systems were approximately linked to the heat transports of fully blocked and unblocked systems in a manner that was linearly dependent on the ratio of the height of the barrier to the depth of the annulus.

(319 words)

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LIST OF PRINCIPAL QUANTITIES USED AND THEIR DEFINITIONS.

Quantity	Definition
A_*	Defined by equation (3.17), $A_*^{-1} \approx H_{adv}/H_\eta$.
a, b	Inner and outer radii of annulus, see Table 2.1.
$a(\bar{r}; \phi)$	Modification to z_0 due to sloping bases, see equation (3.3).
C_p	Specific heat capacity of fluid, see Table 2.1.
$d, (\bar{d})$	Depth (or mean depth) of annulus cavity, see Table 2.1.
E	Subscript 'E' generally denotes a standard error (§2.2.8).
Ek	Ekman number, see equation (1.15).
g	Acceleration due to gravity, 980 cm. sec^{-2} .
Gr	Grashof number, see below equation (1.14).
H_{adv}, \bar{H}_{adv}	Advective heat transport, or flux.
H_{cond}, \bar{H}_{cond}	Conductive heat transport, or flux.
$H_{2/3}, H_{1/3}$	Heat advection above 2/3, or 1/3 partial barrier.
H_η, H_ζ	Heat advection by the η or ζ -circulation.

Quantity	Definition
h	Height of partial barrier.
k	Thermal conductivity of fluid, see Table 2.1.
Nu	Nusselt number, see equation (1.17).
p	Pressure in fluid.
Pr	Prandtl number, see equation (1.16).
(r, ϕ, z)	Coordinates of point in cylindrical polar coordinates.
Ra	Rayleigh number, see equation (1.14).
Ro	Rossby number, see equation (1.12).
$\mathcal{S}, d\vec{\mathcal{S}}$	Surface, or vector element of surface.
slope	Superscript 'slope' denotes the correction to allow for a sloping base.
T, \bar{T}	Temperature, or mean temperature of fluid, see Table 2.1.
T'	Deviation of T from linearity, see equation (3.4).
t	Time.
\vec{u}	Fluid velocity with components (u, v, w) .
u_η	Radial velocity of η -circulation.
u_ζ	Radial velocity of ζ -circulation.
u'	Radial velocity departures from $u_\eta + u_\zeta$.
Y	$\approx \Delta T_B$, see equation (3.20).
z_0, z_1	Integration limits over z , see equation (3.3).
α	Expansion coefficient of fluid, see Table 2.1.
ΔT	Externally applied temperature difference, see Table 2.1.
$\Delta T'$	Range of departures from linearity of $T'(\bar{r}, z = 0; \phi, t)$.
ΔT_B	Temperature drop across barrier.
$\Delta T_r, \Delta T_\phi, \Delta T_z$	Typical temperature differences across body of fluid.
Δu_ϕ	Shear in u over ϕ associated with ζ -circulation.
ϵ	Half angular thickness of barrier.
ζ	-circulation, see §3.2.1.
η	-circulation, see §3.2.1.
Θ	'Theta', see equation (1.18).
κ	Thermometric conductivity of fluid = $k/\rho C_p$.
ν	Kinematic viscosity of fluid, see Table 2.1.
$\rho, \bar{\rho}$	Fluid density, mean fluid density, see Table 2.1.
σ	Subscript ' σ ' denotes a fractional error in a quantity (§2.2.8).
τ	Taylor number, see equation (1.13).
Φ	Potential of external forces.
ϕ_0, ϕ_1	Integration limits over ϕ , see equation (3.3).
$\vec{\Omega}$	Rotation vector for annulus, see Table 2.1.